MATH 112 - SPRING 2011 EXAM 2 ANSWERS

1. (15 Points) Fatal Errors: Simplify each expression as much as possible.

   Show all work for these problems, or you may receive NO CREDIT.

   (a) Simplify the radical:

   $$\sqrt{w^4+9w^2y^2}$$

   $$\sqrt{w^4+9w^2y^2}=\sqrt{w^2(w^2+9y^2)}=\sqrt{w^2}\sqrt{w^2+9y^2}=w\sqrt{w^2+9y^2}$$

   (b) Rewrite without negative exponents or fractions in fractions:

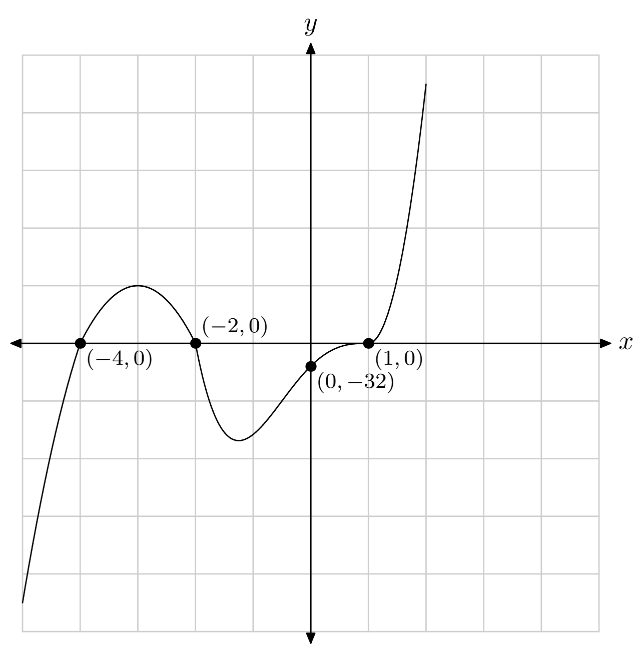
   $$\left(x^{-1}-k^{-1}\right)^{-1}$$

   $$\left(x^{-1}-k^{-1}\right)^{-1}=\left(\frac{1}{x}-\frac{1}{k}\right)^{-1}=\left(\frac{k}{xk}-\frac{x}{xk}\right)^{-1}=\left(\frac{k-x}{xk}\right)^{-1}=\frac{xk}{k-x}$$

   (c) Factor and simplify: $\dfrac{9s^2t^3}{3st^5+6s^3t^2}$

   $$\frac{9s^2t^3}{3st^5+6s^3t^2}=\frac{9s^2t^3}{3st^2(t^3+2s^2)}=\frac{3st}{t^3+2s^2}$$

2. (10 Points) Graph the polynomial $y=4(x+2)(x-1)^3(x+4)$. Label your intercepts and justify your graph with a sign chart or something equivalent.



3. (10 Points) Perform the following long division, and identify the quotient and remainder:

   $$\frac{x^3-x^2+2}{x^2+2x-3}$$

$$
\begin{array}{r}
x - 3 \\
x^2 + 2x - 3 \overline{) \; x^3 \; - x^2 \qquad + 2} \\
- x^3 - 2x^2 + 3x \qquad \\
\hline
- 3x^2 + 3x + 2 \\
3x^2 + 6x - 9 \\
\hline
9x - 7
\end{array}
$$

So the quotient is $x - 3$ and the remainder is $9x - 7$.

4. (15 Points) Let $f(x) = 2x^2 - 4x - 5$.

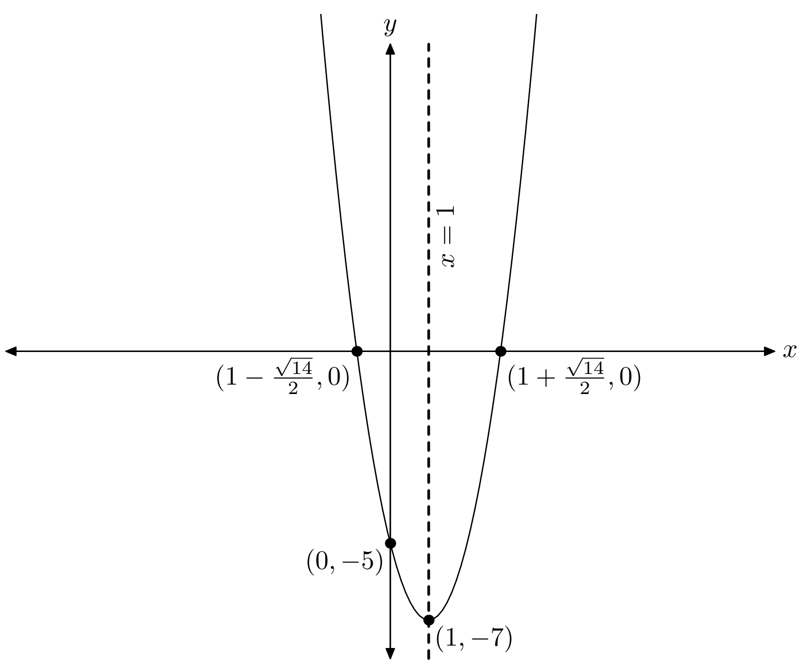
   (a) Find the x- and y-intercepts.

   Setting $x = 0$ gives the y-intercept: $f(0) = 2(0)^2 - 4(0) - 5 = -5$.
   Setting $f(x) = 0$ and solving finds the x-intercepts:
   $2x^2 - 4x - 5 = 0 \implies x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(2)(-5)}}{2(2)} = 1 \pm \frac{\sqrt{56}}{4} = 1 \pm \frac{\sqrt{14}}{2}$.

   (b) Find the coordinates of the vertex and the equation for the axis of symmetry.

   The x-coordinate for the vertex is $h = \frac{-b}{2a} = \frac{-(-4)}{2(2)} = 1$.
   The y-coordinate is $f(1) = 2 - 4 - 5 = -7$.
   The axis of symmetry is $x = 1$.

   (c) Graph $y = f(x)$. Label all intercepts, vertex and axis of symmetry.



5. (20 Points) We intend to enclose a rectangular area by building a wooden fence, using a pre-existing stone wall for one of the sides. The cost of fence for segments perpendicular to the stone wall is \$2 per foot, and the length parallel to the wall costs \$5 per foot. Let x be the length of fence perpendicular to the wall and y the length parallel to the wall. We have a total of \$200 to spend. See the figure.

(a) If we want 20 feet of fence perpendicular to the wall ($x = 20$), and we spend all the money, how long must the length parallel to the wall (y) be? What is the resulting enclosed area?

If $x = 20$, then the cost of the two segments of fence will be 40 feet $\times$ \$2 / foot = \$80. The remaining \$120 will be spent on fence at \$5 per foot, so y would be $120/5 = 24$ feet.
Therefore, the enclosed area would be $20 \times 24 = 480$ feet.

(b) Find a function $A(x)$, which gives the enclosed area of the fence, by spending all \$200.

The total cost of the fence is $5y + 2(2x)$, and with \$200, we get

$$5y + 4x = 200 \implies 5y = 200 - 4x \implies y = 40 - \frac{4}{5}x.$$

So the enclosed area A would be

$$A = xy = x(40 - \frac{4}{5}x) = 40x - \frac{4}{5}x^2 = A(x)$$

(c) Find the dimensions which **maximizes** the enclosed area in this situation. Explain why you know your answer to yield the maximum.

The value of area is determined by a quadratic function for which the leading coefficient is negative ($-\frac{4}{5} = a$). Therefore, the vertex is the point whose input (value of x) will give the largest output (value of A). The x-coordinate of the vertex is

$$\frac{-b}{2a} = \frac{-40}{2(-\frac{4}{5})} = 40 \cdot \frac{5}{8} = 25$$

So when x is 25 feet, the area is maximal. Then

$$y = 40 - \frac{4}{5}(25) = 40 - 20 = 20$$

Therefore, the maximal enclosed area is $A = 25 \times 20 = 500$ square feet.

6. (15 Points) Solve the given inequality. Give your answer in **interval notation**. Show all work.

$$3x^2 \geq 2x + 1$$

$3x^2 \geq 2x + 1 \implies 3x^2 - 2x - 1 \geq 0 \implies (3x + 1)(x - 1) \geq 0$
So $x = -\frac{1}{3}$ and $x = 1$ are our key values:

Interval	Test Value	$3x + 1$	$x - 1$	$(3x + 1)(x - 1)$	≥ 0?
$(-\infty, -\frac{1}{3})$	-1	$\ominus$	$\ominus$	$\oplus$	Yes
$-\frac{1}{3}$		0	$\ominus$	0	Yes
$(-\frac{1}{3}, 1)$	0	$\oplus$	$\ominus$	$\ominus$	No
1		$\oplus$	0	0	Yes
$(1, \infty)$	2	$\oplus$	$\oplus$	$\oplus$	Yes

So the solution is $(-\infty, -\frac{1}{3}] \cup [1, \infty)$.

7. (15 Points) Solve the given inequality. Give your answer in **interval notation**. Show all work.

$$|2-3x| > 7$$

$$\begin{aligned} 2-3x &> 7 \\ -3x &> 5 \\ x &< -\tfrac{5}{3} \end{aligned} \qquad \text{or} \qquad \begin{aligned} 2-3x &< -7 \\ -3x &< -9 \\ x &> 3 \end{aligned}$$

So the solution is $(-\infty, -\frac{5}{3}) \cup (3, \infty)$.

8. (15 Points) Given that -1 is a root of $f(x) = x^3 - x^2 - 5x - 3$, find all solutions to

$$x^3 - x^2 - 5x - 3 = 0.$$

Since -1 is a solution, we know that $x+1$ divides $f(x)$, so using long division (or synthetic division:

$$\begin{array}{r|l} & x^2 - 2x - 3 \\ x+1 & x^3 - x^2 - 5x - 3 \\ & -x^3 - x^2 \\ \hline & -2x^2 - 5x \\ & 2x^2 + 2x \\ \hline & -3x - 3 \\ & 3x + 3 \\ \hline & 0 \end{array}$$

The quotient can be factored as $x^2 - 2x - 3 = (x-3)(x+1)$. So
$0 = x^3 - x^2 - 5x - 3 = (x+1)(x^2 - 2x - 3) = (x+1)(x-3)(x+1) \implies x = -1,\ 3.$

9. (10 Points) Let $f(x) = \dfrac{x^2 - 2x + 3}{4x^2 + 4}$.

(a) Find the domain of $f(x)$, and find all vertical and horizontal asymptotes, if any.

The domain will be all values except those for which
$4x^2 + 4 = 0 \implies 4x^2 = -4 \implies x^2 = -1 \implies \ldots$
So there are no bad values, making the domain be $(-\infty, \infty)$.
Therefore, there are no vertical asymptotes for this rational function.
When x gets big, then

$$f(x) = \frac{x^2 - 2x + 3}{4x^2 + 4} \approx \frac{x^2}{4x^2} = \frac{1}{4}$$

So the graph of this rational function will have a horizontal asymptote of $y = \frac{1}{4}$.

(b) Does the graph $y = f(x)$ ever cross its horizontal asymptote? If so, find the point of intersection. If not, justify your answer.

We find when $f(x) = \frac{1}{4}$:

$$\begin{aligned} \tfrac{1}{4} &= \frac{x^2 - 2x + 3}{4x^2 + 4} \\ x^2 + 1 &= x^2 - 2x + 3 \\ 1 &= -2x + 3 \\ -2 &= -2x \\ x &= 1 \end{aligned}$$

So the graph of $f(x)$ crosses its horizontal asymptote at the point $(1, \frac{1}{4})$.

10. (10 Points) Find the remainder of $\dfrac{x^{49} - 7x^2}{x - 2}$.

By the remainder theorem, the remainder of this would be $f(2)$, where $f(x) = x^{49} - 7x^2$, so the remainder will be $2^{49} - 7(2^2)$.

11. (15 Points) Let $f(x) = \sqrt{x+1}$ and $g(x) = x^2 - 1$. Show all work on this problem.

(a) Find $(f \circ g)(x)$ and its domain.

$(f \circ g)(x) = f(g(x)) = f(x^2 - 1) = \sqrt{(x^2 - 1) + 1} = \sqrt{x^2} = |x|$.
The domain of this function is $(-\infty, \infty)$.

(b) Find $(g \circ f)(x)$ and its domain.

$(g \circ f)(x) = g(f(x)) = g(\sqrt{x+1}) = (\sqrt{x+1})^2 - 1 = x + 1 - 1 = x$.
The domain of this function is $[-1, \infty)$.

(c) Are f and g inverses? Justify your answer.

No, their compositions don't both give x.