

Syllabus for Math 234 Lecture 2, Spring 2009 - A. Assadi (Instructor); George Brown, Alec Johnson, Fang Yuan (TA)
Instructor Remarks on Chapter 14, 14.4 – 14.6

Lectures Tuesday Feb 10, and Thursday Feb 12, 2009 will cover 14.4 – 14.6.

Reading Before Coming to the Lectures: You must have read and know the materials in Section 14.1-14.4 , pp 965-998

NOTES. I have discussed partial derivatives of $f(x,y)$ with respect to x and y as special cases of directional derivative. Then I defined the gradient and directional derivatives in general. In class, we will do examples to understand the definitions and concepts better.

The definitions of tangent planes to surfaces that are graphs of functions of two variables and also parametric surfaces were mentioned intuitively. We will learn the rigorous definition and we will do examples of calculations in class.

I would recommend that you overview the equations and graphs of several familiar surfaces that we call S (such as a cylinder through the line $y=x$ on the (x,y) -plane and of radius R , a cone whose axis is the y -axis and has the tip angle = 60 degrees, a sphere of radius R centered at the origin, an ellipsoids with principal diameters $= (a,b,c)$; a paraboloid of revolution using the z -axis as the revolution axis and the curve $z = x^2$ in the (x,z) -plane; and as many different representatives of hyperboloids of one sheet or two sheets that you could think of and make them have a symmetric form relative the z -axis, and make the z -axis go through the points that remain fixed when you rotate the hyperboloid to show some of its symmetries.) Some of these were discussed earlier in class, and others are in Section 14.1 or elsewhere in the textbook.

Try to write down the parametric equation of a piece of the surface as a vector function of two variables $\mathbf{X}(u,v) = (x(u,v), y(u,v), z(u,v))$ where (u,v) belong to the domain of definition of \mathbf{X} . For the domain of definition, take the points inside of a disk of radius R (choose R numerically and judiciously) or a Square centered at the origin of side $2a$.

Choose constant values of $(u, v) = (a, b)$, for example, for the sphere, choose (a, b) such that the point on S is the north pole of the sphere. Then find the point $\mathbf{P} = \mathbf{X}(a,b)$. Find the coordinate curves lying on such surfaces that pass through the point \mathbf{P} . by setting first $v = b$ (the number b is a constant that **you** choose), and write down the equation of the curve \mathbf{C}_1 that is given by the vector equation $\mathbf{X}(u, b)$ and u is the curve-parameter. Then find the tangent vector to \mathbf{C}_1 at \mathbf{P} . Repeat the same steps for the curve \mathbf{C}_2 that is given by the vector equation $\mathbf{X}(a, v)$ and v is the curve-parameter. Pay special attention to the curves and the formulas that you obtain for the tangent vectors. Use the tangent vectors to calculate the vector that is **Normal to the Surface S** , and then use this normal vector to write the equation of the tangent plane to S at \mathbf{P} . I will do an example to illustrate these steps in class. But you must do many examples, such as the ones above, to feel comfortable with the entire process and do them fairly quickly.

Please read the examples in the text, and make sure you remember all the facts that Math 221 had covered for Chain Rule.

I will be also covering Taylor's Formula for $f(x, y)$ near a point (x_0, y_0) . Using the linear terms in this Taylor expansion, we can find the formula for the Tangent Plane.

Conversely, we can use information about the tangent at a point P to the surface S that is the graph of an unknown function $f(x, y)$, and calculate the linear terms of the Taylor expansion of $f(x, y)$ near the point P. Suppose that S is given as the graph of an unknown function $f(x, y)$, and we are given only the geometric structure of the surface by the set of points on S (that is, if you choose a point on S, the computer provides you with the coordinates of the point. Therefore, we have the coordinates of the point P on S, where the tangent plane is defined. Use the above-mentioned surfaces and the equation of their Tangent Plane at P to the surface S to calculate the Taylor expansion of the function $f(x, y)$ whose graph defines S.

NOTES. Please review change of variables, polar coordinates and differentiation rules for Implicit Functions, Inverse Functions and the Chain Rule from Calculus 221.

EVERYONE is expected to know the formulas, be able to do medium-level exercises right away in class on the Prerequisite materials from earlier courses.

Please look up the key words above from the Index of the text-book, go directly to the pages, and check out from the first few examples if you are on top of the required calculations.

If some of you have difficulty in the background, make sure you get help by going to MATH LAB on the second floor, or the tutorial service (must be a group of three or more to get help.)

Textbook's Try the easiest and some medium-numbered (e.g. Odd-numbered exercises in the sections that we cover as mentioned above). Then test your skills on the review and advanced exercises on pages 1060-1061 and pp 1063-1064 (the Extremum problems are for the following week.)