MATH 234 (A. Assadi) Spring 2009 Final Exam (Practice)

**Instructions.** (1) This test is CLOSED BOOK. You are allowed to have two index-cards or one page of notebook paper with formulas only. Calculators are allowed ONLY for computation and graphing.

(2) The time is 120 minutes for working on the test.

(3) All functions and vector fields are assumed to be differentiable everywhere, unless otherwise stated.

*We rely on honor system for the students to use only their own work and without asking from or offering help to other students. Please Observe the Honor System.*

**Time: 120 minutes**

**YOUR NAME:** **TA NAME:**

**SECTION:**

| Problem | Points | Score |
|---|---|---|
| 1 | | |
| 2 | | |
| 3 | | |
| 4 | | |
| 5 | | |
| 6 | | |
| 7 | | |
| 8 | | |
| 9 | | |

NAME (print): ______________________________

**Problem 1.** The plane $x+y+4z=2$ cuts the cone $z^2=x^2+y^2$ in an ellipse. Using the method of Lagrange multipliers, find the greatest and the smallest values that the function $f(x,y,z)=z^2$ takes on the ellipse. Write the equations that you need to solve in the box below. Put your final answer in the two boxes at the bottom.

Greatest value: $\boxed{\phantom{xxxx}}$ Smallest value: $\boxed{\phantom{xxxx}}$

NAME (print): ______________________________

**Problem 2.**

(a) Compute the gradient of the function $f(x, y, z) = e^x y + \cos y + 2zx$.

(b) Let $\mathbf{F} = e^z\,\mathbf{i} + \sin y\,\mathbf{j} + 4xyz\,\mathbf{k}$. Find $\mathrm{curl}(\nabla f + \mathbf{F})$.

(c) Now let $\mathbf{G} = e^x y\,\mathbf{i} + \sin y\,\mathbf{j} + x^2 x\,\mathbf{k}$. Compute $\mathrm{div}\,(\mathbf{G} + \mathrm{curl}(\nabla f + \mathbf{F}))$.

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**Problem 3.** Consider the annular ring $A = \{1 \leqslant x^2 + y^2 \leqslant 4\}$ and let $M = y(1 - \cos(xy))$, and $N = x(1 + \cos(xy)$.

(a) Sketch the region $A$ and the vector field $\mathbf{F} = M\,\mathbf{i} + N\,\mathbf{j}$ at the points where the boundary of $A$ intersects the coordinate axes. Make sure that your vectors have proper length!

(b) Compute the circulation

$$\int_{\partial A} M\,dx + N\,dy$$

of $\mathbf{F}$ around the boundary of $A$. Hint: Green's theorem might be useful.

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**Problem 4.** Consider the sphere $x^2+y^2+z^2=a^2$ and the cylinder $x^2+y^2=a^2$. Let $0 \leqslant h_1 < h_2 \leqslant a$. Show that the surface area of the part of the sphere between the planes $z=h_1$ and $z=h_2$ equals the surface area of the part of the cylinder between the same two planes. In other words, the surface area of the sphere over any vertical span is equal to the surface area of the circumscribing cylinder over that same span. (This was already know to Archimedes!)

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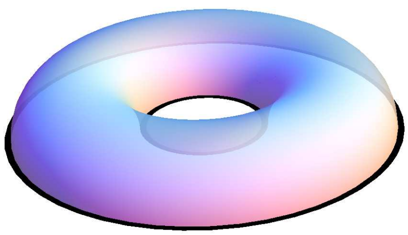
**Problem 5.** Consider the surface $S$, which is the upper half of the torus (see figure) and whose parameterization is

$$\mathbf{r}(u,v) = \langle (2+\cos u)\cos v, (2+\cos u)\sin v, \sin u \rangle, \quad 0 \leqslant u \leqslant \pi,\ 0 \leqslant v \leqslant 2\pi.$$



Let $\mathbf{F} = \left\langle -y, x, e^{z^2}\cos(x^2) \right\rangle$. Compute

$$\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{n}\, d\sigma,$$

where $\mathbf{n}$ is the upward normal.

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**Problem 6.** Compute the outward flux of

$$\mathbf{F} = 2x\,\mathbf{i} + 3y\,\mathbf{j} - 4z\,\mathbf{k}$$

across the boundary of the upper half of the solid unit sphere, that is, across the boundary of the set

$$D = \left\{(x, y, z) :\ x^2 + y^2 + z^2 \leqslant 1,\ z \geqslant 0\right\}.$$