Problem: Expand the function

$$f(x,y) = y^2 e^{x+y} + x^3 y^2 + 5$$

in a Taylor series centered around the origin $(0,0)$ out to fourth order.

Solution: We want a polynomial whose derivatives agree with the derivatives of f near the origin. One way to do this is to calculate all the required derivatives.

There is a shortcut. This is nearly a polynomial already. The one problem is $g(x,y) := e^{x+y}$. If we replace this function with its fourth-order Taylor series we will be done (once we have expanded and collected like terms). But here we can make use of the Taylor series for the one-variable function

$$e^t = 1 + t + t^2/2 + t^3/3! + t^4/4! + \cdots .$$

We plug in $x + y$ for t and get $\ldots$.