Viaclovsky HW 1 (Due Sep 10)

§1.1

(7) $y'' - 2y' + 2y = 0$

(2) Show $y_1 = e^x \cos x$
and $y_2 = e^x \sin x$
are solutions.

Solution:

y_1: $2\,[\,y_1 = e^x \cos x\,]$
$-2\,[\,y_1' = e^x(\cos x - \sin x)\,]$
$y_1'' = e^x(-2\sin x)$

$y_1'' - 2y_1' - 2y = 0$ ✓

y_2: $2\,[\,y_2 = e^x \sin x\,]$
$-2\,[\,y_2' = e^x(\sin x + \cos x)\,]$
$y_2'' = e^x(2\cos x)$

$y_2'' - 2y_2' - 2y = 0$ ✓

(10) $x^2 y'' + x y' - y = \ln x$

(2) $y_1 = x - \ln x$
$y_2 = \frac{1}{x} - \ln x$
Show y_1 & y_2 are solutions.

y_1: $-\,[\,y_1 = x - \ln x\,]$
$+x\,[\,y_1' = 1 - \frac{1}{x}\,]$
$+x^2\,[\,y_1'' = \frac{1}{x^2}\,]$

$x^2 y_1'' + x y' - y_1 = \ln x$ ✓

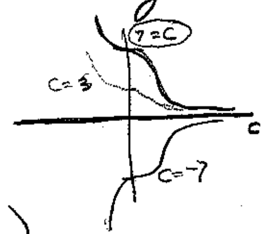
y_2: $-1\,[\,y_2 = \frac{1}{x} - \ln x\,]$
$x\,[\,y_2' = \frac{-1}{x^2} - \frac{1}{x}\,]$
$x^2\,[\,y_2'' = \frac{2}{x^3} + \frac{1}{x^2}\,]$

$x^2 y_2'' + x y_2' - y_2 = \ln x$ ✓

(21) $y' + 3x^2 y = 0$
(4) $y(x) = Ce^{-x^3}$
$y(0) = 7$
Check y, find C.

Graph solutions, hilight for IC

$y' = -C \cdot 3x^2 e^{-x^3} = -3x^2 y$ ✓
$y(0) = \boxed{C = 7}$

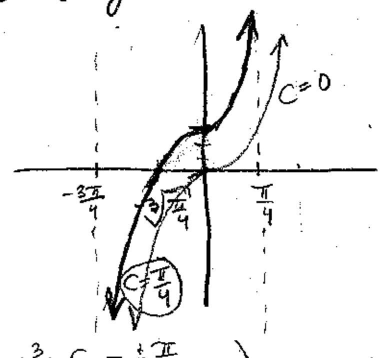


(25) $y' = 3x^2(y^2+1)$
(4) $y(x) = \tan(x^3 + C)$
$y(0) = 1$ (IC = initial condition)
Check y, find C. Graph.

$y' = \sec^2(x^3 + C) \cdot 3x^2$
$=$ RHS (right-hand side)
using that $\tan^2\theta + 1 = \sec^2\theta$

To find C plug general solution into IC:
$1 = y(0) = \tan(C)$,
So $\boxed{C = \arctan 1 = \frac{\pi}{4}}$ works.

Graphing:
Can use slope field of ODE to help graph.
Can use level sets of slope function
$f(x,y) = 3x^2(y^2+1)$ to create slope field.
For vertical asymptotes set argument of tangent to $\pm\frac{\pi}{2}$



$\left(\begin{array}{l} x^3 + C = \pm\frac{\pi}{2} \\ \text{says} \quad x = \sqrt[3]{\pm\frac{\pi}{2} - C} \end{array} \right)$

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Solution:

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$$\begin{aligned} 2[\,y_1 &= e^x \cos x\,] \\ -2[\,y_1' &= e^x(\cos x - \sin x)\,] \\ y_1'' &= e^x(-2 \sin x) \end{aligned}$$

$$y_1'' - 2y_1' - 2y = 0 \ \checkmark$$

y_2:
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$$x^2 y_1'' + x y' - y_1 = \ln x \ \checkmark$$

y_2:
$$\begin{aligned} -1[\,y_2 &= \tfrac{1}{x} - \ln x\,] \\ x[\,y_2' &= \tfrac{-1}{x^2} - \tfrac{1}{x}\,] \\ x^2[\,y_2'' &= \tfrac{2}{x^3} + \tfrac{1}{x^2}\,] \end{aligned}$$

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$$y' = -C \cdot 3x^2 e^{-x^3} = -3x^2 y \ \checkmark$$

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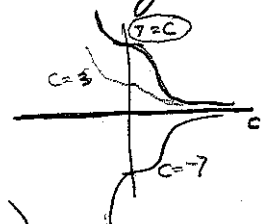

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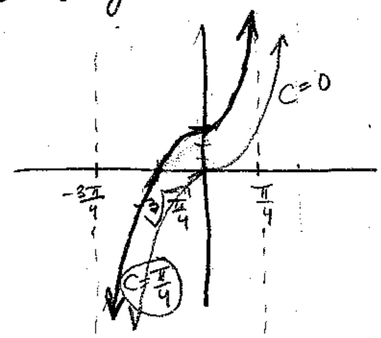

$$\left(\begin{aligned} & x^3 + C = \pm\tfrac{\pi}{2} \\ \text{says} \quad & x = \sqrt[3]{\pm\tfrac{\pi}{2} - C} \end{aligned} \right)$$

Viadovsky HW1

§1.2

(6) $\frac{dy}{dx} = x\sqrt{x^2+9}$

(2) $y(-4) = 0.$

Find $y(x)$.

Solution:

$y = \int x\sqrt{x^2+9}\,dx$

$\boxed{y = \frac{1}{3}\sqrt{x^2+9}^3 + C}$

$y(-4) = \frac{1}{3}\sqrt{4^2+9}^3 + C$

$0 = \frac{1}{3}5^3 + C$

$\boxed{C = \frac{-5^3}{3} = \frac{-125}{3} = -(41+\tfrac{2}{3})}$

(28) $V_0 = {}^-40$ (ft/s)

$X_0 = 555$ (ft)

(2) $v' = a;$

$a = -32$ ft/s²

Find:

(a) time t_f to reach ground

(b) speed $V_f = v(t_f)$.

$x'' = v' = a$

$x' = v = at + V_0 \quad = -32t - 40$

$x = \frac{1}{2}at^2 + V_0 t + X_0 = -16t^2 - 40t + 555$

$0 = \frac{1}{2}at_f^2 + V_0 t_f + X_0 = -16t_f^2 - 40t_f + 555$

Solve for t_f.

$t_f = \frac{-V_0 \pm \sqrt{V_0^2 - 2aX_0}}{a}$

$= \frac{40 \pm \sqrt{40^2 + 2\cdot 32\cdot 555}}{-32}.$

$\boxed{t_f \cong 4.77s}$ (choosing positive solution.)

$V_f = at_f + V_0 = -32t_f - 40$

$\boxed{V_f \cong -192.6 \text{ ft/s}}$