Helena Menendez
Math 320:324
11/10/15

Homework #10: Wednesday

§5.1

10) $y'' - 10y' + 25y = 0$ $\qquad y_1 = e^{5x},\ y_2 = xe^{5x};\ y(0)=3;\ y'(0)=13$

a)

y_2: $\cancel{25xe^{5x}} + \cancel{5e^{5x}} - \cancel{50xe^{5x}} - \cancel{10e^{5x}} + \cancel{25xe^{5x}} + \cancel{5e^{5x}} = 0$

y_1: $\cancel{25e^{5x}} - \cancel{50e^{5x}} + \cancel{25e^{5x}} = 0$

	y	y'	y''
y_1	e^{5x}	$5e^{5x}$	$25e^{5x}$
y_2	xe^{5x}	$5xe^{5x} + e^{5x}$	$25xe^{5x} + 5e^{5x} + 5e^{5x}$

yes, they both are sol'ns.

9/9

b) $y(x) = c_1e^{5x} + c_2xe^{5x}$

$y'(x) = 5c_1e^{5x} + 5c_2xe^{5x} + c_2e^{5x}$

$y(0) = c_1 = 3$

$y'(0) = 5c_1 + c_2 = 13,\ c_2 = -2$

so: $\boxed{y = 3e^{5x} - 2xe^{5x}}$

12) $y'' + 6y' + 13y = 0$ $\qquad y_1 = e^{-3x}\cos 2x,\ y_2 = e^{-3x}\sin 2x,\ y(0)=2,\ y_1'(0)=0$

a)

y_1: $9e^{-3x}\cos 2x + 6e^{-3x}\sin 2x + 6e^{-3x}\sin 2x - 4e^{-3x}\cos 2x - 18e^{-3x}\cos 2x - 12e^{-3x}\sin 2x + 13e^{-3x}\cos 2x = 0$ ✓

y_2: $9e^{-3x}\sin 2x - 6e^{-3x}\cos 2x - 6e^{-3x}\cos 2x - 4e^{-3x}\sin 2x - 18e^{-3x}\sin 2x + 12e^{-3x}\cos 2x + 13e^{-3x}\sin 2x = 0$ ✓

yes, both are sol'ns!

b) $y(x) = c_1e^{-3x}\cos 2x + c_2e^{-3x}\sin 2x$

$y'(x) = -3c_1e^{-3x}\cos 2x - 2c_1e^{-3x}\sin 2x - 3c_2e^{-3}\sin 2x + 2c_2e^{-3x}\cos 2x$

$y(0) = c_1 = 2$

$y'(0) = -3c_1 + 2c_2 = 0;\ c_2 = 3$

so: $\boxed{y(x) = 2e^{-3x}\cos 2x + 3e^{-3x}\sin 2x}$

written as "$e^{-3x}(2\cos 2x + 3\sin 2x) = y$" in book

§5.2

8) $f(x)=e^{x}$, $f'(x)=e^{x}$, $f''(x)=e^{x}$
$g(x)=e^{2x}$; $g'(x)=2e^{2x}$; $g''(x)=4e^{2x}$
$h(x)=e^{3x}$; $h'(x)=3e^{3x}$; $h''(x)=9e^{3x}$

$$W(f,g,h)=\begin{vmatrix} e^{x} & e^{2x} & e^{3x} \\ e^{x} & 2e^{2x} & 3e^{3x} \\ e^{x} & 4e^{2x} & 9e^{3x} \end{vmatrix}$$

$$=e^{x}(18e^{5x}-12e^{5x})-e^{x}(9e^{5x}-4e^{5x})+e^{x}(3e^{5x}-2e^{5x})$$

$$=6e^{6x}-5e^{6x}+e^{6x}=2e^{6x}\neq 0$$ yes, they're lin. indep.!

14) $y^{(3)}-6y''+11y'-6y=0$ $\quad y(0)=0$; $y'(0)=0$; $y''(0)=3$
$y_1=e^{x}$; $y_2=e^{2x}$; $y_3=e^{3x}$

$y(x)=c_1e^{x}+c_2e^{2x}+c_3e^{3x}$
$y'(x)=c_1e^{x}+2c_2e^{2x}+3c_3e^{3x}$
$y''(x)=c_1e^{x}+4c_2e^{2x}+9c_3e^{3x}$

$$\left.\begin{aligned} y(0)&=c_1+c_2+c_3=0\\ y'(0)&=c_1+2c_2+3c_3=0\\ y''(0)&=c_1+4c_2+9c_3=3\end{aligned}\right\} \left[\begin{array}{ccc|c} 1&1&1&0\\1&2&3&0\\1&4&9&3\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1&1&1&0\\0&1&2&0\\0&3&8&3\end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1&1&1&0\\0&1&2&0\\0&0&1&3/2\end{array}\right]$$

$$\Rightarrow \left[\begin{array}{ccc|c} 1&0&0&3/2\\0&1&0&-3\\0&0&1&3/2\end{array}\right]$$

So: $\boxed{y(x)=\tfrac{3}{2}e^{x}-3e^{2x}+\tfrac{3}{2}e^{3x}}$ written as $y(x)=\tfrac{1}{2}(3e^{x}-6e^{2x}+3e^{3x})$ in the book

#24 on reverse →

20) $f(x) = 2\cos x + 3\sin x$; $f'(x) = -2\sin x + 3\cos x$
$g(x) = 3\cos x - 2\sin x$; $g'(x) = -3\sin x - 2\cos x$

$$W(f,g) = \begin{vmatrix} f & g \\ f' & g' \end{vmatrix} = \begin{vmatrix} 2\cos x + 3\sin x & 3\cos x - 2\sin x \\ -2\sin x + 3\cos x & -3\sin x - 2\cos x \end{vmatrix}$$

$$= -6\cos x\sin x - 4\cos^2 x - 9\sin^2 x - \cancel{6\cos x \sin x} - (\cancel{-6\cos x\sin x} + 9\cos^2 x + 4\sin^2 x - \cancel{6\cos x\sin x})$$

$$= -4\cos^2 x - 9\sin^2 x - 9\cos^2 x - 4\sin^2 x$$

$$= -13(\cos^2 x + \sin^2 x) = \emptyset$$

so: they are lin. indep because the determinant is not zero

34) $y'' + 2y' - 15y = 0$ Recall: $y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$ (Thm 5)

$r^2 + 2r - 15 = (r+5)(r-3) = 0$

$r = -5, 3$

so $\boxed{y(x) = c_1 e^{-5x} + c_2 e^{3x}}$

40) $9y'' - 12y' + 4y = 0$

$9r^2 - 12r + 4 = 0$ $\qquad \dfrac{+12 \pm \sqrt{144 - 4(9)(4)}}{2(9)} = \dfrac{12 \pm \sqrt{0}}{18} = \dfrac{2}{3}$

check $(3r-2)(3r-2) = 0$

so $r_1 = 2/3$ Recall: $y(x) = (c_1 + c_2 x)e^{r_1 x}$ (Thrm 6)

so: $\boxed{y(x) = c_1 e^{2/3 x} + c_2 x e^{3/2 x}}$

46) $y(x) = c_1 e^{10x} + c_2 e^{100x}$ $\qquad r_1 = 10,\ r_2 = 100$

$y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x}$

$(r-10)(r-100)$

$r^2 - 110r + 1000$

$\boxed{y'' - 110y' + 1000y = 0}$

24) $y'' - 2y' + 2y = 2x$ $\qquad$ $y(0) = 4,\ y'(0) = 8$

$y_c = c_1 e^x \cos x + c_2 e^x \sin x$

$y_p = x + 1$

so: $y_1 = e^x \cos x$, $y_2 = e^x \sin x$

$y = y_c + y_p$

$y = c_1 e^x \cos x + c_2 e^x \sin x + x + 1$

$y' = c_1 e^x \cos x - c_1 e^x \sin x + c_2 e^x \sin x + c_2 e^x \cos x + 1$

$y(0) = c_1 + 1 = 4 \quad \Rightarrow c_1 = 3$

$y'(0) = c_1 + c_2 + 1 = 8 \Rightarrow c_2 = 4$

so: $\boxed{y = 3e^x \cos x + 4e^x \sin x + x + 1}$