HW 12

5.4: ~~4~~, 13, ~~14~~
5.5: ~~4~~, ~~8~~, 10, 32, ~~34~~, 35

6.1: 10, 20, 30

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TA: Alec

4a) $m = 1/4$ kg, $k = (9N)/(0.25m) = 36$ N/m
$\omega_0 = 12$ rad/sec, $x'' + 144x = 0$, $x(0) = 1$ and $x'(0) = -5$
is $x(t) = \cos 12t - (5/12)\sin 12t$
$= (13/12)[(12/13)\cos 12t - (5/13)\sin 12t]$
$x(t) = (13/12)\cos(12t - \alpha)$
where $\alpha = 2\pi - \tan^{-1}(5/12) \approx 5.8884$

b) $C = 13/12 \approx 1.0833$ m and $T = 2\pi/12 \approx 0.5236$ sec

13/13

13a) The eq. $10r^2 + 9r + 2 = (5r+2)(2r+1) = 0$
roots: $r = -2/5, -1/2$, impose $x(0) = 0$, $x'(0) = 5$
$x(t) = c_1 e^{-2t/5} + c_2 e^{-t/2}$ we get soln. $x(t) = 50(e^{-2t/5} - e^{-t/2})$

b) $x'(t) = 25e^{-t/2} - 20e^{-2t/5} = 5e^{-2t/5}(5e^{-t/10} - 4) = 0$
when $t = 10\ln(5/4) = 2.23144$.
$x(10\ln(5/4)) = 512/125 = 4.096$

14)a) $25r^2 + 10r + 226 = (5r+1)^2 + 15^2 = 0$, $r = (-1 \pm 15i)/5 = -1/5 \pm 3i$
$x(0) = 20$, $x'(0) = 41$, gen soln: $x(t) = e^{-t/5}(A\cos 3t + B\sin 3t)$
$A = 20$, $B = 15$, particular $x(t) = e^{-t/5}(20\cos 3t + 15\sin 3t) =$
$25e^{-t/5}\cos(3t - \alpha)$ where $\alpha = \tan^{-1}(3/4) \approx 0.6435$

b) Ocillations bounded by curves $x = \pm 25e^{-t/5}$
and psuedoperiod is $T = 2\pi/3$ cuz $\omega = 3$

5.5 4) $4y'' + 4y' + y = 3xe^x$
$y_{trial} = Ae^x + Bxe^x$
$9A + 12B = 0$, $9B = 3$

$y_p = (-4e^x + 3xe^x)/9$

6) $2y'' + 4y' + 7y = x^2$

$y_{trial} = A + Bx + Cx^2$ $7A + 4B + 4C = 0$, $7B + 8C = 0$ $7C = 1$

$y_p = (4 - 56x + 49x^2)/343$

8) $y'' - 4y = \cosh 2x$ complementary $y_c = c_1 \cosh 2x + c_2 \sinh 2x$

~~th~~ then $y_{trial} = x(A \cosh 2x + B \sinh 2x)$ $4A = 0$, $4B = 1$

$y_p = (1/4) x \sinh 2x$

10) $y'' + 9y = 2\cos 3x + 3\sin 3x$ comp. $y_c = c_1 \cos 3x + c_2 \sin 3x$

$y_{trial} = x(A\cos 3x + B \sin 3x)$

$6B = 2$, $-6A = 3$ $y_p = (2x \sin 3x - 3x \cos 3x)/6$

33) $y'' + 3y' + 2y = e^x$ $y(0) = 0$, $y'(0) = 3$

$y_c = c_1 e^{-x} + c_2 e^{-2x}$ $y_{trial} = Ae^x$

$y_g = c_1 e^{-x} + c_2 e^{-2x} + e^x/6$

$c_1 + c_2 + 1/6 = 0$, $-c_1 - 2c_2 + 1/6 = 3$

$y(x) = (15 e^{-x} - 16 e^{-2x} + e^x)/6$

34) $y'' + y = \cos x$, $y(0) = 1$, $y'(0) = 0$

$y_c = c_1 \cos x + c_2 \sin x$ $y_{trial} = x \cdot (A \cos x + B \sin x)$

$y_g = c_1 \cos x + c_2 \sin x + \frac{1}{2} x \sin x$

$c_1 = 1$ $c_2 = -1$ $y(x) = \cos x - \sin x + \frac{1}{2} x \sin x$

35) $y'' - 2y' + 2y = x + 1$, $y(0) = 3$, $y'(0) = 0$

$y_c = e^x(c_1 \cos x + c_2 \sin x)$; $y_{trial} = A + Bx$

$y_g = e^x(c_1 \cos x + c_2 \sin x) + 1 + x/2$

$c_1 + 1 = 3$, $c_1 + c_2 + 1/2 = 0$

$y(x) = e^x(4\cos x - 5 \sin x)/2 + 1 + x/2$

6.1 10) $\begin{bmatrix} 9 & -10 \\ 2 & 0 \end{bmatrix}$ poly: $p(\lambda) = \lambda^2 - 9\lambda + 20 = (\lambda - 4)(\lambda - 5)$

eigen: $\lambda_1 = 4$ $\lambda_2 = 5$

$\lambda_1 = 4$: $\left.\begin{array}{l} 5a - 10b = 0 \\ 2a - 4b = 0 \end{array}\right\}$ $v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

$\lambda_2 = 5$: $\left.\begin{array}{l} 4a - 10b = 0 \\ 2a - 5b = 0 \end{array}\right\}$ $V_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

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20) $\begin{bmatrix} 1 & 0 & 0 \\ -4 & 7 & 2 \\ 10 & -15 & -4 \end{bmatrix}$ $\quad p(\lambda) = -\lambda^3 + 4\lambda^2 - 5\lambda + 2 = -(\lambda-1)^2(\lambda-2)$

eigens: $\lambda_1 = \lambda_2 = 1 \quad \lambda_3 = 2$

$\lambda_1 = 1$: $\left.\begin{array}{l} 0 = 0 \\ -4a + 6b + 2c = 0 \\ 10a - 15b - 5c = 0 \end{array}\right\}$ $v_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \quad v_2 = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$\lambda_3 = 2$: $\left.\begin{array}{l} -a = 0 \\ -4a + 5b + 2c = 0 \\ 10a - 15b - 5c = 0 \end{array}\right\}$ $v_3 = \begin{bmatrix} 0 \\ -2 \\ 5 \end{bmatrix}$

30) $A = \begin{bmatrix} 0 & -12 \\ 12 & 0 \end{bmatrix}$ $\quad p(\lambda) = \lambda^2 + 144$

eigens: $\lambda_1 = -12i \quad \lambda_2 = +12i$

$\lambda_1 = -12i$ $\left.\begin{array}{l} 12ia - 12b = 0 \\ 12a + 12ib = 0 \end{array}\right\}$ $v_1 = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

$\lambda_2 = +12i$: $\begin{array}{l} -12ia - 12b = 0 \\ 12a - 12ib = 0 \end{array}$ $v_2 = \begin{bmatrix} i \\ 1 \end{bmatrix}$