1.3 - 22 | 1.4 - 3 12 22 48 | 1.5 8 14 16 31 32 37

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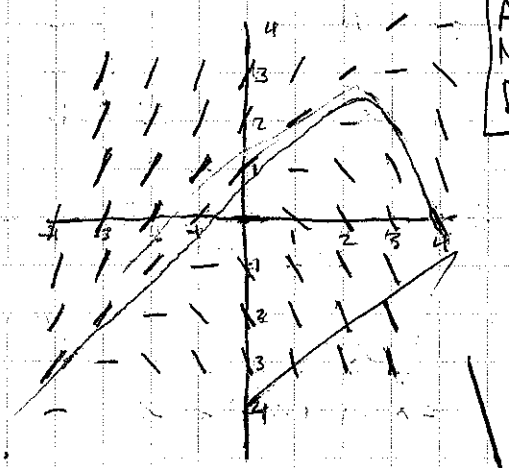
22)

y \ x	-3	-2	-1	0	1	2	3
-3	0	-1	-2	-3	-4	-5	-6
-2	1	0	-1	-2	-3	-4	-5
-1	2	1	0	-1	-2	-3	-4
0	3	2	1	0	-1	-2	-3
1	4	3	2	1	0	-1	-2
2	5	4	3	2	1	0	-1
3	6	5	4	3	2	1	0



$y(-4) \approx -3$

1.4

3) $\frac{dy}{dx} = y\sin x \Rightarrow \int \frac{dy}{y} = \int \sin x\,dx \Rightarrow \ln|y| = -\cos x + C$

$\boxed{y = C'e^{-\cos x}}$

12) $yy' = x(y^2+1)$ $\quad \int \frac{y\,dy}{y^2+1} = \int x\,dx \Rightarrow \frac{\ln|y^2+1|}{2} = \frac{x^2}{2} + C$

$y^2+1 = C'e^{x^2}$ $\quad \boxed{y = \pm(C'e^{x^2}-1)^{1/2}}$

22) $\int \frac{dy}{y} = \int 4x^3 - 1\,dx$ $\quad \ln|y| = x^4 - x + C$ $\quad y = C'e^{x^4-x}$ $\quad C' = -3$

$\boxed{y = -3e^{(x^4-x)}}$

48) $U_{238} = U_0 2^{-t/T_{1/2}}$ $\quad \frac{U_{238}}{U_{235}} = \frac{U_{0238}}{U_{0235}} 2^{-t/T_{238} - (-t/T_{235})}$ $\quad -t\left(\frac{1}{T_{238}} - \frac{1}{T_{235}}\right)$

same @ beginning $\Rightarrow \frac{^{238}U_0}{^{235}U_0} = 1$

$\Rightarrow 137.7 = 2^{-t\left(\frac{1}{T_{235}} - \frac{1}{T_{238}}\right)}$

$\frac{-\left(\frac{\ln(137.7)}{\ln 2}\right)}{\left[\left(\frac{1}{T_{238}}\right) - \left(\frac{1}{T_{235}}\right)\right]} = t$

$\boxed{t = 5.98 \times 10^9 \text{ years}}$

1.5 ~~8~~ ~~14~~ ~~16~~ ~~32~~ 37

8) $y' + \frac{y}{3x} = 4$ $\quad \rho = e^{\int \frac{1}{3x}} = e^{\ln x^{1/3}} = x^{1/3}$

$(x^{1/3}y)' = 4x^{1/3}$ $\quad yx^{1/3} = 3x^{4/3} + C$ $\quad \boxed{y = 3x + \frac{C}{x^{1/3}}}$

14) $y' - \frac{3}{x}y = x^3$ $\quad \rho = e^{\int -3/x} = x^{-3}$

$(x^{-3}y)' = \frac{1}{x}$ $\quad x^{-3}y = \ln|x| + C$ $\quad y = x^3 \ln x + x^3 C \Big|_{x=1} = 10;\ C = 10$

$\Rightarrow \boxed{y = x^3 \ln x + 10x^3}$

16) $y' = (1-y)\cos x = \cos(x) + \cos(x)y$ $\quad C = 9.75$

$y' + \cos(x)y = \cos(x)$ $\quad \rho = e^{\int \cos x} = e^{\sin x}$

$(y \cdot e^{\sin(x)})' = \cos x\, e^{\sin x}$

$y \cdot e^{\sin(x)} = e^{\sin x} + C$ $\quad y = 1 + \frac{C}{e^{\sin(x)}} \Big|_{x=\pi} = 2;\ C = 1$

$\Rightarrow \boxed{y = 1 + e^{-\sin(x)}}$

31) a) $y' = -cPe^{-\int P}$ $\Rightarrow$ $y' + P(x)y = 0$ $\Rightarrow$ $-cPe^{-\int P} + Pce^{-\int P} = 0$ true

b) $y = e^{-\int P}\int Qe^{\int P}$ $\quad y' = \left[-Pe^{-\int P}\int Qe^{\int P} + e^{-\int P}Qe^{\int P}\right]$

$\rightarrow [\ \ ] + Pe^{-\int P}\int Qe^{\int P} = e^{-\int P}Qe^{\int P} = Qe^0 = Q$ true

c) $y(x) = ce^{-\int P} + e^{-\int P}\int Qe^{\int P}$ $\quad y'(x) = \left[-Pce^{-\int P} - Pe^{-\int P}\int Qe^{\int P} + e^{-\int P}Qe^{\int P}\right]$

$[\ \ ] + Pce^{-\int P} + Pe^{-\int P}\int Qe^{\int P} = Q$ true

32) a) $y(x) = A\sin x + B\cos x$ $\quad y' + y - 2\sin x = (A-B)\sin x + (A+B)\cos x = 2\sin x$

$y' = A\cos x - B\sin x$

$\rightarrow \boxed{\sin x - \cos x = y}$ $\quad A - B = 2 \rightarrow A = 1$, $A + B = 0 \rightarrow B = -1$

b) $y' + y = 0$ $\quad y' = -y$ $\quad y = ce^{-x}$ $\quad y = ce^{-x} + \sin x - \cos x$

c) $y = ce^{-x} + \sin x - \cos x \Big|_{x=0} = 1;\ c = 2$

$\boxed{y = 2e^{-x} + \sin x - \cos x}$

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37) $V_f = 400$ gal $V_0 = 100$ gal
$x_f = ?$ $x_0 = 50$ lbs
$c_i = 1$ lb/gal $r_i = 5$ gal/s
$r_o = 3$ gal/s

$x' = \overset{5}{r_i} c_i - \overset{3}{r_o} c(t)$

$c(t) = \frac{x(t)}{v(t)}$

$v(t) = 100 + 2t$

$x' = 5 - \frac{3x(t)}{100+2t}$ $\quad x' + \frac{3x(t)}{100+2t} = 5$ $\quad \rho = e^{\int \frac{3}{100+2t}} = e^{\frac{3}{2}\ln|50+t|} = (50+t)^{3/2}$

$[(50+t)^{3/2} x(t)]' = 5(50+t)^{3/2}$

$(50+t)^{3/2} x(t) = 5\int (50+t)^{3/2}\, dt = 2(50+t)^{5/2} + C$

$x(t) = 2(50+t) + \frac{C}{(50+t)^{3/2}} = 100 + 2t + \frac{C}{(50+t)^{3/2}}$

$V(t) = 400$ @ $t = 150$

$C = -17678$

$x(t) = 100 + 2t + \frac{-17678}{(50+t)^{3/2}}\Big|_{t=150} = 393.75$ lbs