$\frac{11}{11}$

1.6 (6, 10, 18, 22, 34, 36, 37) 2.1 (5, 6, 21, 28)

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sec: 323

$\boxed{6.}$ (homogeneous)

$$V = \frac{y}{x}$$
$$y = vx$$
$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\frac{(x+2y)}{x} y' = \frac{y}{x}$$

$$\left(1 + \frac{2y}{x}\right) y' = \frac{y}{x}$$

$$y' = \frac{\frac{y}{x}}{1 + \frac{2y}{x}}$$

$$v + x\frac{dv}{dx} = \frac{v}{1+2v}$$

$$x\frac{dv}{dx} = \frac{v}{1+2v} - v$$

$$\frac{dV}{\frac{v}{1+2v} - v\left(\frac{1+2v}{1+2v}\right)} = \frac{1}{x}dx$$

$$\frac{dv}{\frac{\cancel{v-v}-2v^2}{1+2v}} = \frac{1}{x}dx$$

$$\frac{1+2v}{-2v^2}\,dv = \frac{1}{x}dx$$

$$\int -\frac{1}{2v^2}dv - \int \frac{1}{v}dv = \int \frac{1}{x}dx$$

$$\frac{1}{2v} - \ln|v| = \ln|x| + c$$

$$\frac{1}{2\frac{y}{x}} - \ln\left|\frac{y}{x}\right| = \ln|x| + c$$

$$\frac{x}{2y} - \ln|y| + \cancel{\ln|x|} = \cancel{\ln|x|} + c$$

$$\frac{x}{2y} - \ln|y| = c$$

$$\boxed{x - 2y\ln|y| = yc'}$$

$\boxed{10.}$ (homogeneous) $\boxed{V = \frac{y}{x}}$

$$\frac{xy}{x^2} y' = \frac{x^2}{x^2} + \frac{3y^2}{x^2}$$

$$\frac{y}{x} y' = 1 + 3\left(\frac{y}{x}\right)^2$$

$$v\left(v + x\frac{dv}{dx}\right) = 1 + 3v^2$$

$$v + x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$x\frac{dv}{dx} = \frac{1}{v} + 2v$$

$$dv = \left(\frac{1}{v} + 2v\left(\frac{v}{v}\right)\right)\frac{1}{x}dx$$

$$dv = \left(\frac{1+2v^2}{v}\right)\frac{1}{x}dx$$

$$\int \frac{v}{1+2v^2}dv = \int \frac{1}{x}dx \qquad u = 1+2v^2 \quad du = 4v\,dv$$

$$\frac{1}{4}\int \frac{1}{u}du = \int \frac{1}{x}dx$$

$$\frac{1}{4}\ln|1+2v^2| = \ln|x| + c$$

$$\frac{1}{4}\ln|1+2v^2| - \frac{1}{4}\ln|x|^4 = c$$

$$\cancel{4\frac{1}{4}}\ln\left|\frac{1}{x^4} + \frac{2v^2}{x^4}\right| = c \cdot 4$$

$$e^{\frac{1}{4}\ln}\left|\frac{1+2v^2}{x^4}\right| = c' \cdot e$$

$$\frac{1+2v^2}{x^4} = c''$$

$$\left(\frac{x^2}{x^2}\right)\frac{1}{x^4} + \frac{2y^2}{x^6} = c''$$

$$\frac{x^2 + 2y^2}{x^6} = c''$$

$$\boxed{x^2 + 2y^2 = x^6 c''}$$

$\boxed{18}$ $(x+y)y' = 1$ (substitution)

$z = x+y$

$y = z-x$

$\frac{dy}{dx} = \frac{dz}{dx} - 1$

$$z\left(\frac{dz}{dx}-1\right) = 1$$

$$\frac{dz}{dx} - 1 = \frac{1}{z}$$

$$\frac{dz}{dx} = \frac{1}{z} + 1$$

$$\frac{dz}{dx} - \frac{1}{z} = 1$$

$$\rho = e^{-\int 1\,dx} = e^{-x}$$

$$\int \frac{d}{dx}\left(\frac{1}{z}e^{-x}\right) = \int e^{-x}\,dx$$

$$\frac{1}{z}\cdot e^{-x} = -e^{-x} + c$$

$$\frac{1}{(x+y)}\cdot e^{-x} = -e^{-x} + c$$

~~$\ln|x| \cdot \ln|y| \cdot -\ln|e^{x}| = -\ln|e| + c'$~~

$$-x\ln|x+y| = -x + c$$

$$-x\ln|x+y| + x = c$$

$$\boxed{x(-\ln|x+y| + 1) = c}$$

$\boxed{22}$ $\frac{x^2y'}{x^2} + \frac{2xy}{x^2} = \frac{5y^4}{x^2}$

$$y' + \frac{2y}{x} = \frac{5y^4}{x^2}$$

$$v = y^{1-4} = \frac{1}{y^3}$$

$$y = v^{-\frac{1}{3}}$$

(Bernoulli)

$$\frac{dy}{dx} = -\frac{1}{3}v^{-\frac{4}{3}}$$

$$\cancel{v^{4/3}}\left(-\frac{1}{\cancel{3v^{4/3}}}\frac{dv}{dx} + \frac{2}{xv^{1/3}}\right) = \left(\frac{5}{x^2\cancel{v^{4/3}}}\right)\cancel{v^{4/3}}$$

$$-\cancel{3}\cdot\left(-\cancel{1/3}\frac{dv}{dx} + \frac{2v}{x} = \frac{5}{x^2}\right)$$

$$\frac{dv}{dx} - \frac{6v}{x} = -\frac{15}{x^2}$$

$$\rho = e^{\int -\frac{6}{x}dx} = e^{-6\ln|x|} = x^{-6}$$

$$\int \frac{d}{dx}\left(\frac{1}{x^6}v\right) = \int -\frac{15}{x^8}\,dx$$

$$\frac{v}{x^6} = \frac{15}{7x^7} + c$$

$$\left(\frac{1}{x^6}\right)\left(\frac{1}{y^3}\right) = \frac{15}{7x^7} + c$$

$$\frac{1}{y^3} = \frac{15}{7x} + x^6c\left(\cdot\frac{7x}{7x}\right)$$

$$\frac{1}{y^3} = \frac{15 + 7x^7c}{7x}$$

$$\boxed{y^3 = \frac{7x}{15 + 7x^7c}}$$

$$\boxed{34}\ \overset{(m)}{(2xy^2+3x^2)}dx + \overset{(N)}{(2x^2y+4y^3)}dy = 0$$

$$\frac{\partial M}{\partial y} = 4yx \qquad \frac{\partial N}{\partial x} = 4xy$$

$$\frac{\partial M}{\partial x} = \frac{\partial N}{\partial x} \rightarrow \text{true}$$

$$\frac{\partial E}{\partial x} = M = \int 2xy^2 + 3x^2\,dx = x^2y^2 + x^3 + g(y)$$

$$\frac{\partial F}{\partial y} = N_{given} = 2x^2y + 4y^3\,dy$$

$$\frac{\partial N}{\partial x} = 4xy$$

$$\int \partial N = \int 4xy\,\partial x$$

$$N_{calculated} = 2x^2y$$

$$(N_{calculated} = N_{given})$$

$$\cancel{2x^2y} = \cancel{2x^2y} + 4y^3$$

$$\frac{\partial g}{\partial y} = 4y^3$$

$$\int \partial g = \int 4y^3\,\partial y$$

$$g(y) = y^4 + c$$

$$\boxed{\text{answer: } F = x^2y^2 + x^3 + y^4 + c}$$

$$\boxed{36}\ (\overset{M}{1+ye^{xy}})dx + (\overset{N}{2y+xe^{xy}})dy = 0$$

$$\frac{\partial M}{\partial y} = y\cdot xe^{xy} + e^{xy} = \frac{\partial N}{\partial x} = xye^{xy} + e^{xy}$$

true

$$M = \frac{\partial F}{\partial x} = 1 + ye^{xy}$$

$$\int \partial F = \int ye^{xy} + 1\,\partial x$$

$$F = e^{xy} + x + g(x)$$

integrate by parts

$dv = ye^{xy}dx$

$u = x$

$v = e^{xy}$

$du = dx$

$$\frac{\partial N}{\partial x} = xye^{xy} + e^{xy}$$

$$\int \partial N = \int xye^{xy} + e^{xy}\,\partial x$$

$$N = \frac{1}{y}e^{xy} + xe^{xy} - \int e^{xy}dx$$

$$N = \cancel{\frac{1}{y}e^{xy}} + xe^{xy} - \cancel{\frac{1}{y}e^{xy}}$$

$$N_{calculated} = xe^{xy}$$

$$N_{calc.} = N_{given}$$

$$\cancel{xe^{xy}} = 2y + \cancel{xe^{xy}}$$

$$\int \partial g = \int 2y\,dy$$

$$g(y) = y^2 + c$$

$$\boxed{\text{answer: } F = e^{xy} + x + y^2 + c}$$

$\boxed{37}$ $\overset{M}{(\cos x + \ln y)}dx + \overset{N}{\left(\frac{x}{y} + e^y\right)}dy = 0$

$\frac{\partial M}{\partial y} = \frac{1}{y} = \frac{\partial N}{\partial x} = \frac{1}{y}$

true

$\frac{\partial N}{\partial x} = \frac{1}{y}$

$\int \partial N = \int \frac{1}{y}\, \partial x$

$N_{calc} = \frac{x}{y}$

$N_{calc.} = N_{given}$

$\cancel{\frac{x}{y}} = \cancel{\frac{x}{y}} + e^y$

$\int \frac{\partial g}{\partial y} = \int e^y dy$

$g(y) = e^y + c$

$M = \frac{\partial F}{\partial x} = \cos x + \ln y$

$\int \partial F = \int \cos x + \ln y\, \partial x$

$F = \sin x + x\ln|y| + g(x)$

$\boxed{\text{answer: } F = \sin x + x\ln|y| + e^y + c}$

$\boxed{2.1}$ $\boxed{5}$ $\frac{dx}{dt} = 3x(5-x)$, $x(0) = 8$

$(-)\frac{1}{x(5-x)}dx = (-)3t$

$-\frac{1}{x(x-5)}dx = -3t\, dt$

$\cancel{(-5)}\left(\frac{1}{x} - \frac{1}{x-5}\right)\left(\cancel{\cdot -\frac{1}{5}}\right)\cancel{dx} -3t \quad dt\,(-5)$

$\int \frac{1}{x} - \frac{1}{x-5}\, dx = \int 15\, dt$

$\ln|x| - \ln|x-5| = 15t + c$

$e \cdot \left(\ln\left|\frac{x}{x-5}\right|\right) = (15t+c) \cdot e$

$\frac{x}{x-5} = C' e^{15t}$

$\boxed{x(0)=8} \rightarrow \frac{8}{8-5} = C' \cancel{e^{15(0)}}^{1}$

$c' = \frac{8}{3}$

$\frac{x}{x-5} = \frac{8}{3}e^{15t}$

$\frac{3x}{8x-40} = e^{15t}$

$3x = 8xe^{15t} - 40e^{15t}$

$x(3 - 8e^{15t}) = -40e^{15t}$

$\left(\frac{x+5}{x+5}\right)\frac{A}{x} + \frac{B}{x-5}\left(\frac{x}{x}\right) \rightarrow Ax + A5 - Bx$

$Ax - Bx = 0 \rightarrow A = B$

$5A = 1 \rightarrow A = \frac{1}{5}$

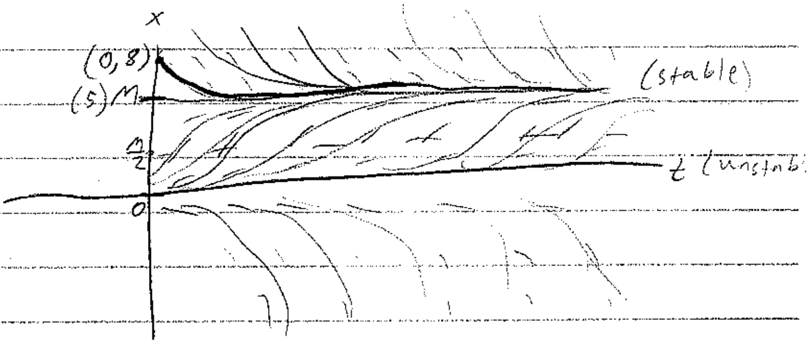
so $A = \frac{1}{5} = B$

$x = \frac{-40e^{15t}}{(3-8e^{15t})} \rightarrow x = \frac{40}{8-3e^{15t}}$

$M = 5$ $P_0 = 8$

$\frac{40}{8-3e^{15t}} \rightarrow \frac{M\cancel{P_0}}{\cancel{P_0}} = M$

plot
(0,8)



⑥ $\frac{dx}{dt} = 3x(x-5),\ x(0) = 2$

$\frac{1}{x(x-5)}\,dx = 3dt$

$\left(\frac{x-5}{x-5}\right)\frac{A}{x} + \frac{B}{(x-5)}\left(\frac{x}{x}\right)$

$Ax - 5A + Bx$

$Ax + Bx = 0$

$A = -B$

$-5A = 1$

$A = \frac{-1}{5}$

$A = -\frac{1}{5}$

$B = \frac{1}{5}$

$\cancel{-5 \times -\frac{1}{5}}\left(\frac{-1}{x} + \frac{1}{(x-5)}\right)dx = 3dt\ (\cdot -5)$

$\int \frac{-1}{x} + \frac{1}{(x-5)}\,dx = \int -15\,dt$

$-\ln|x| + \ln|x+5| = -15t + c$

$-\ln\left|\frac{x-5}{x}\right| = -15t + c$

$e\cdot \ln\left|\frac{x-5}{x}\right| = (15t + c')\cdot e$

$\frac{x-5}{x} = c''e^{15t}$

$\boxed{x(0) = 2} \rightarrow \frac{2-5}{2} = c''e^{\cancel{15(0)}}$

$\boxed{-\frac{3}{2} = c''}$

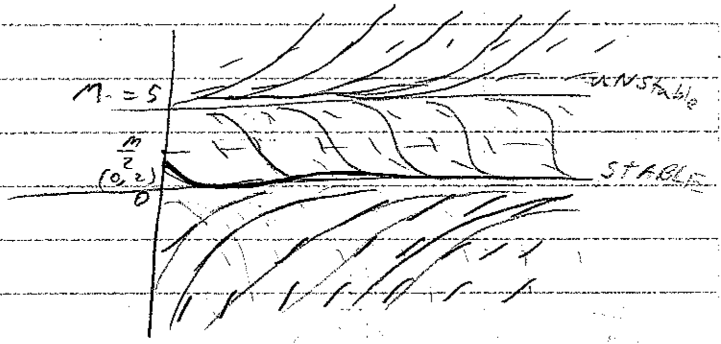
$\frac{x-5}{x} = -\frac{3}{2}e^{15t}$

$2x - 10 = -3xe^{15t}$

$2x + 3xe^{15t} = 10$

$x(2 + 3e^{15t}) = 10$

$\boxed{x = \frac{10}{2+3e^{15t}}}$



$M_i = 5$ $\frac{10}{2+3e^{15t}}$ $<$ $\frac{M_i \cancel{P_0}}{\cancel{P_0}} = M$

$P_0 = 2$

$\boxed{2}$ $\frac{dP}{dt} = KP(\overset{(M)}{200} - P)$

$t(0) = 100$ million
t_0

rate @ t_0 $= 1$ mill/yr

at time $= 60 = ?$ (# people)

$$\frac{dP}{P(200-P)} = K\,dt$$

$$\frac{dP}{P(200-P)} = -K\,dt$$

$$\left(\frac{1}{P} + \frac{1}{200-P}\right) \cdot \frac{1}{200}\,dP = -K\,dt$$

$$\frac{(200+P)}{(200+P)} \cdot \frac{A}{P} + \frac{B}{(200+P)}\left(\frac{P}{P}\right)$$

$$(A+B)P = -200A$$

$A + B = 0$ $\quad -200A = 1$

$A = -B$ $\quad A = -\frac{1}{200}$

$B = -\frac{1}{200}$

$$\int \frac{1}{P} - \frac{1}{200-P}\,dP = \int 200K\,dt$$

$$\ln|P| - \ln|200-P| = 200Kt + c$$

$$e \cdot \ln\left|\frac{P}{200-P}\right| = (200Kt + c) \cdot e$$

$$\frac{P}{200-P} = C'e^{200Kt}$$

$\boxed{P(0) = 100} \rightarrow$ $\frac{100}{200-100} = C'\cancel{e^{200K(0)}} \rightarrow \boxed{C' = 1}$

① $\frac{P}{200-P} = e^{200Kt}$

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② $\frac{dP}{dt} = KP(200-P)$

$\boxed{\text{rate: } t_0 = 1\text{ mill/yr}} \rightarrow$ $1 = K(100)(200-100) \rightarrow \boxed{K = \frac{1}{10000}}$

③ equation: $\frac{P}{200-P} = e^{(200)(\frac{1}{10000})t}$

$$P = 200e^{(200)(\frac{1}{10000})t} - P \cdot e^{(200)(\frac{1}{10000})t}$$

$\boxed{t = 60} \rightarrow$ $P = 200e^{(200)(\frac{1}{10000})(60)} - Pe^{(200)(\frac{1}{10000})(60)}$

$$\boxed{\cancel{P \approx 153.7 \text{ million people}}}$$

$\boxed{28}$ $\frac{dx}{dt} = .0001x^2 - .01x$

ⓐ $.01x(.01x - 1)$

$r(0) = 25$

$\left(\frac{x}{x}\right)\left(\frac{.01x-1}{.01x-1}\right)\frac{A}{x} + \frac{B}{(.01x-1)} \quad \left(\frac{x(.01x-1)}{x(.01x-1)}\right)$

$.01Ax - A + Bx$

$.01Ax + Bx = 0 \rightarrow \boxed{.01 = B}$

$-A = 1 \rightarrow \boxed{A = -1}$

$\cancel{(.01)} \cdot \frac{dx}{\cancel{.01}x(.01x-1)} = dt \cdot (.01)$

$\frac{1}{x(.01x-1)} = .01dt$

$-\frac{1}{x} + \frac{.01}{(.01x-1)}dx = .01dt$

$\int -\frac{1}{x} + \frac{1}{x-100}dx = \int .01dt$

$-\ln|x| + \ln|x-100| = \frac{t}{100} + c$

$e \cdot \ln\left|\frac{x-100}{x}\right| = \left(\frac{t}{100} + c\right) \cdot e$

$\frac{x-100}{x} = c'e^{t/100}$

$\boxed{x(0) = 25}$

$\frac{25-100}{25} = c'\cancel{e^{0/100}}^{1} \rightarrow \boxed{c' = -3}$

$\frac{x-100}{x} = -3e^{t/100}$

$x - 100 = -3x^{t/100}$

$x(1 + 3e^{t/100}) = 100$

$\boxed{x = \frac{100}{1+3e^{t/100}}}$

$\lim_{t\to\infty}\left(\frac{100}{1+3e^{t/100}}\right) = \boxed{0 \text{ alligators die out}}$

ⓑ $x(0) = 150$ general equation: $\frac{x-100}{x} = c'e^{t/100}$

$\frac{150-100}{150} = c'\cancel{e^{0/100}}^{1} \rightarrow \boxed{c' = \frac{1}{3}}$

$\frac{x-100}{x} = \frac{1}{3}e^{t/100}$

$x - 106 = \frac{1}{3}xe^{t/100}$

$x\left(1 - \frac{1}{3}e^{t/100}\right) = 100$

$\boxed{x = \frac{100}{\left(1 - \frac{1}{3}e^{t/100}\right)}} \rightarrow \lim_{t\to\infty}\left(\frac{100}{1-\frac{1}{3}e^{t/100}}\right) = \boxed{\text{explosion}}$

when?: $\frac{x-100}{x} = \frac{1}{3}e^{t/100} \rightarrow \ln\left(\frac{3x-300}{x}\right) = \left(e^{t/100}\right) \cdot \ln$

$\rightarrow t = 100\ln\left|\frac{3x-300}{x}\right| \rightarrow t = 100\ln\left|3 - \frac{300}{x}\right|$

$\lim_{x\to\infty} 100\ln\left|3 - \frac{300}{x}\right| \approx$ 109.86 mo. ≈ 9 yrs, 2 mo.

Doomsday