Fast Magnetic Reconnection in Isotropic Pair Plasma

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Abstract

Question: **What are the minimal modeling conditions that admit fast magnetic reconnection?**

Claim: **Fast magnetic reconnection occurs in an *isotropic* model of pair plasma.** [1]

Therefore the *inertial* term of Ohm's law is sufficient to provide for fast reconnection without the aid of the Hall term or pressure anisotropy.

[1] We use a collisionless two-fluid model, which is not consistent with the assumption of isotropic pressure.

Background

Plasma: gas of charged particles.

Since charged particles spiral around magnetic field lines, *magnetic field lines are approximately frozen into the plasma*.

The topology of magnetic field lines can change only if magnetic field is able to "reconnect", i.e. to cancel or diffuse.

Importance:

1. Change of topology allows plasma to flow to new places.
2. Cancellation of magnetic field releases lots of energy.

Evolution of magnetic field

The evolution of magnetic field is governed by two equations:

① Faraday's law: $\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0$

② Ohm's law (current balance solved for $\mathbf{E}$):

$$
\begin{aligned}
\mathbf{E} = \ & \eta \cdot \mathbf{J} && \text{(resistive term)} \\
& + \mathbf{B} \times \mathbf{u} && \text{(ideal term)} \\
& + \frac{\tilde{m}_i - \tilde{m}_e}{\rho} \mathbf{J} \times \mathbf{B} && \text{(Hall term)} \\
& + \frac{1}{\rho} \nabla \cdot (\tilde{m}_e \mathbb{P}_i - \tilde{m}_i \mathbb{P}_e) && \text{(pressure term)} \\
& + \frac{\tilde{m}_i \tilde{m}_e}{\rho} \Big(\partial_t \mathbf{J} + \nabla \cdot \big(\mathbf{uJ} + \mathbf{Ju} + \frac{\tilde{m}_e - \tilde{m}_i}{\rho} \mathbf{JJ} \big) \Big) && \text{(inertial term).}
\end{aligned}
$$

(Here $\tilde{m}_i := m_i/e, \tilde{m}_e = m_e/e$.)

Theory

What modeling conditions do not admit magnetic reconnection?

① **Existence of a *flux-transporting flow*:** If there exists a velocity field $\mathbf{v}$ for which $\partial_t \mathbf{B} + \nabla \times (\mathbf{B} \times \mathbf{v}) = 0$, then magnetic flux is convected by $\mathbf{v}$ and the topology of magnetic field lines cannot change. In particular, if we merely add the Hall term to the ideal Ohm's law, then $\partial_t \mathbf{B} + \nabla \times (\mathbf{B} \times (\mathbf{u} + \frac{\tilde{m}_e - \tilde{m}_i}{\rho} \mathbf{J} \times \mathbf{B}))$, i.e., the magnetic field is essentially carried by the electrons.

② **Isotropic/gyrotropic pressure:** For isotropic pressures the pressure term of Ohm's law is a gradient (i.e. has zero curl) and therefore is absent from the evolution equation for the magnetic field. (More generally, the divergence of a *gyrotropic* pressure tensor is zero.)

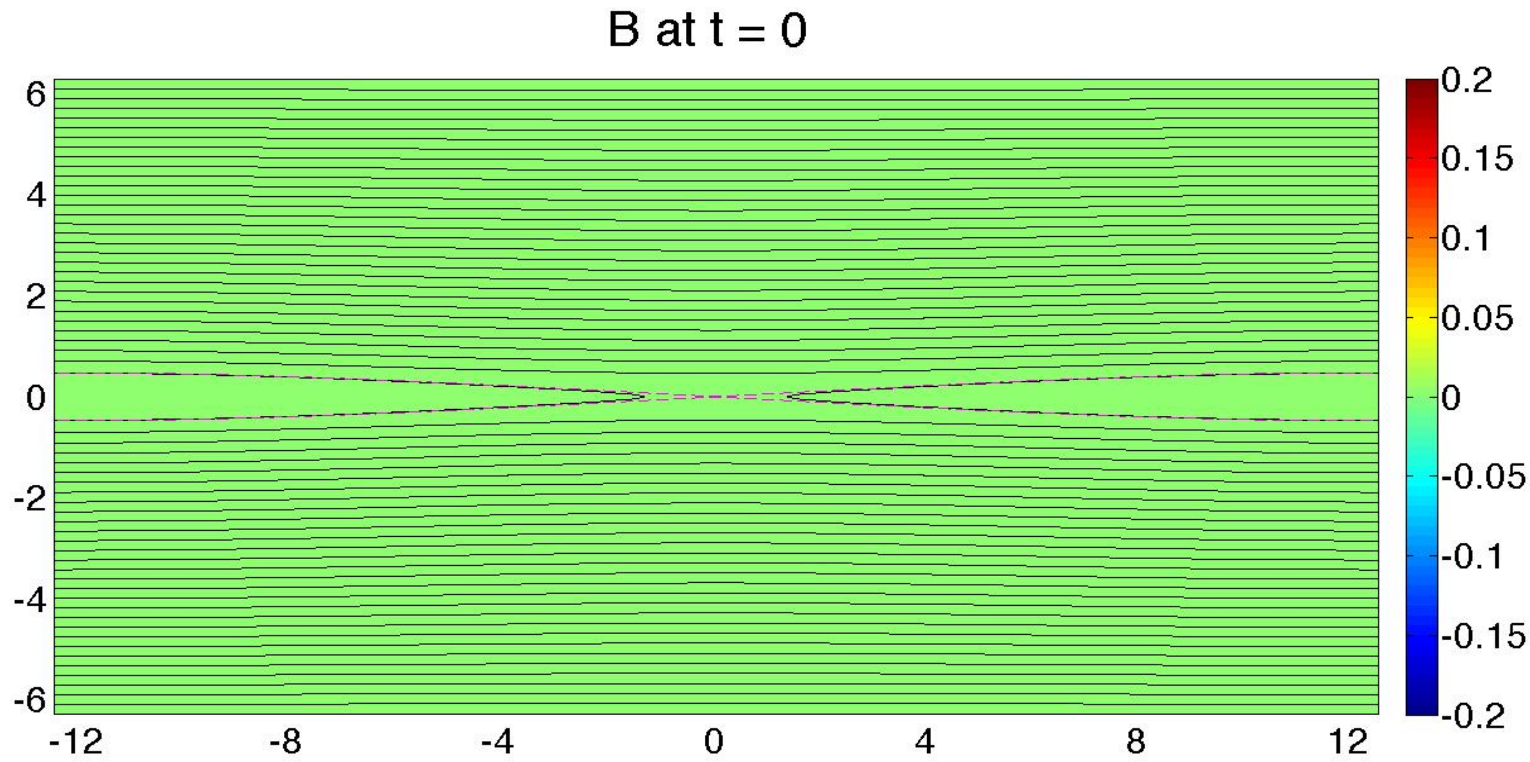
Previous work

What modeling conditions admit fast reconnection?

Model	**Reconnection?**	**Features**
Ideal MHD	no reconnection	
Ideal MHD + resistivity	slow reconnection	Sweet-Parker configuration
Ideal MHD + anomalous resistivity	fast reconnection	no quadrupole structure
Hall MHD	no reconnection	
Hall MHD + small resistivity	fast reconnection	X-configuration, quadrupole
Pair plasma, anisotropic pressure	fast reconnection	X-configuration, no quadrupole

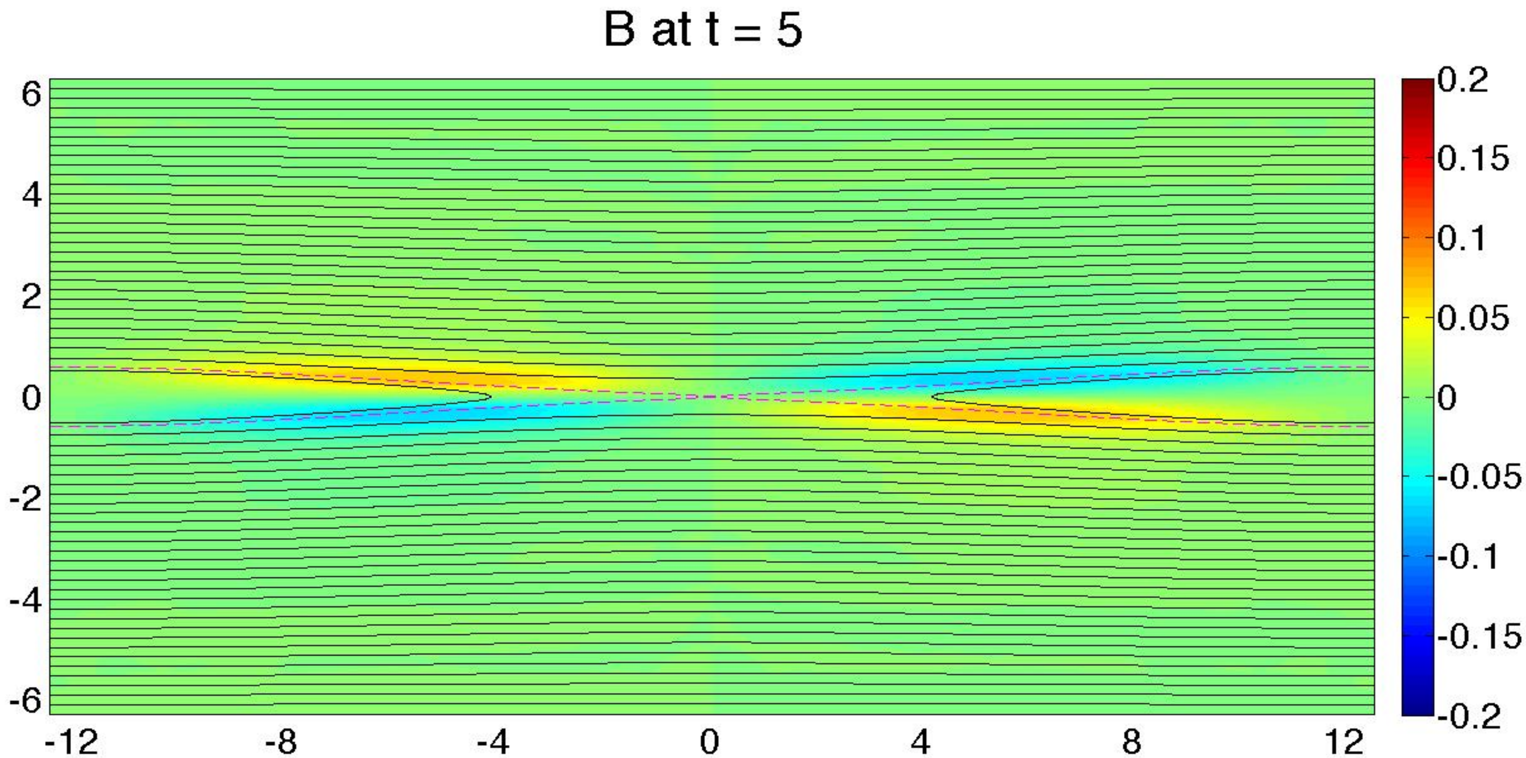
Note that although Hall MHD theoretically does not admit reconnection, the Hall term serves as a *catalyst* which in combination with even a small amount of (possibly numerical) resistivity gives fast reconnection.

GEM magnetic reconnection challenge

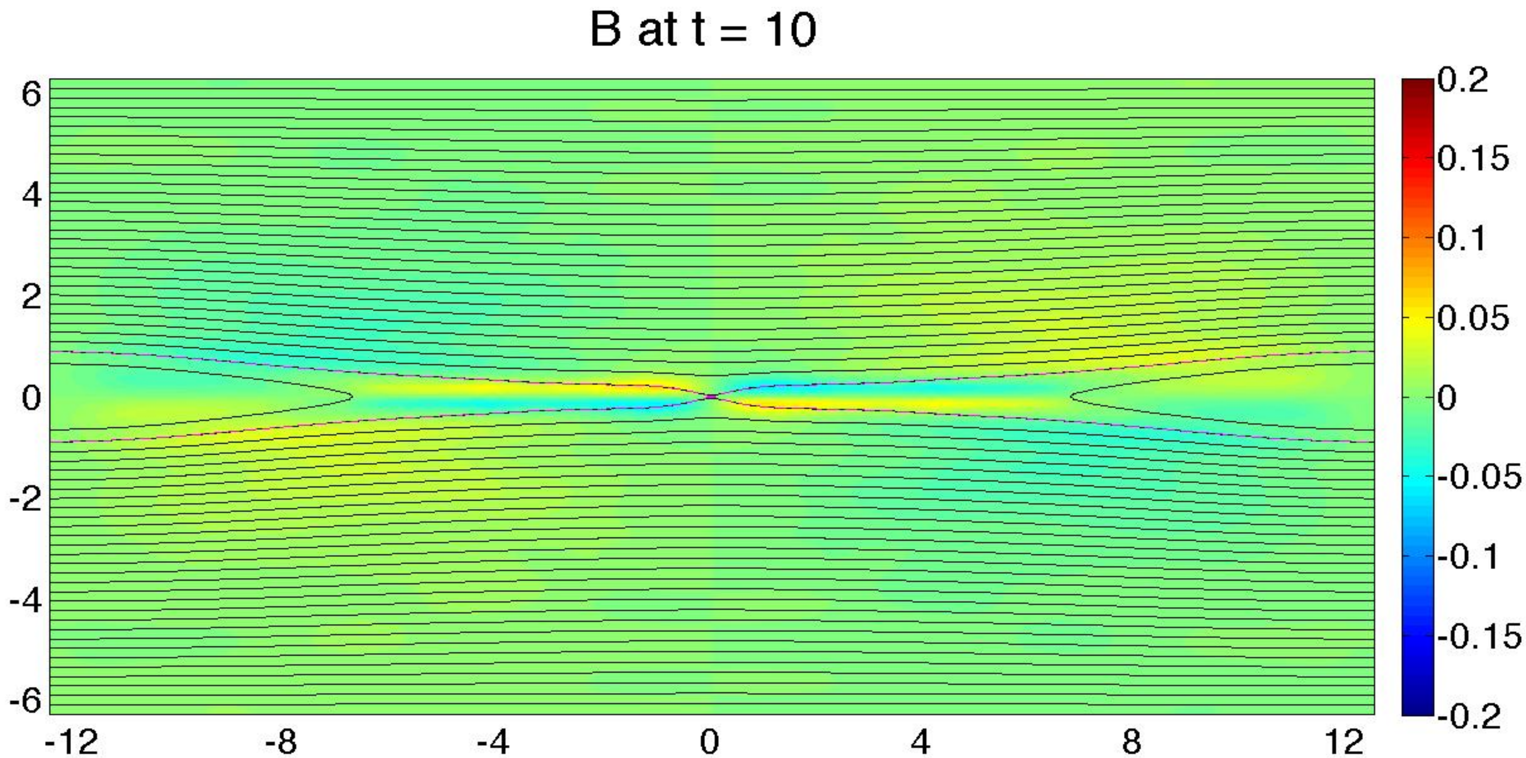


- Initial conditions: Harris sheet equilibrium, perturbed (pinched) $\mathbf{B}$.
- Boundary conditions: conducting walls above and below, horizontally periodic.
- $m_i/m_e = 25, T_i/T_e = 1.$

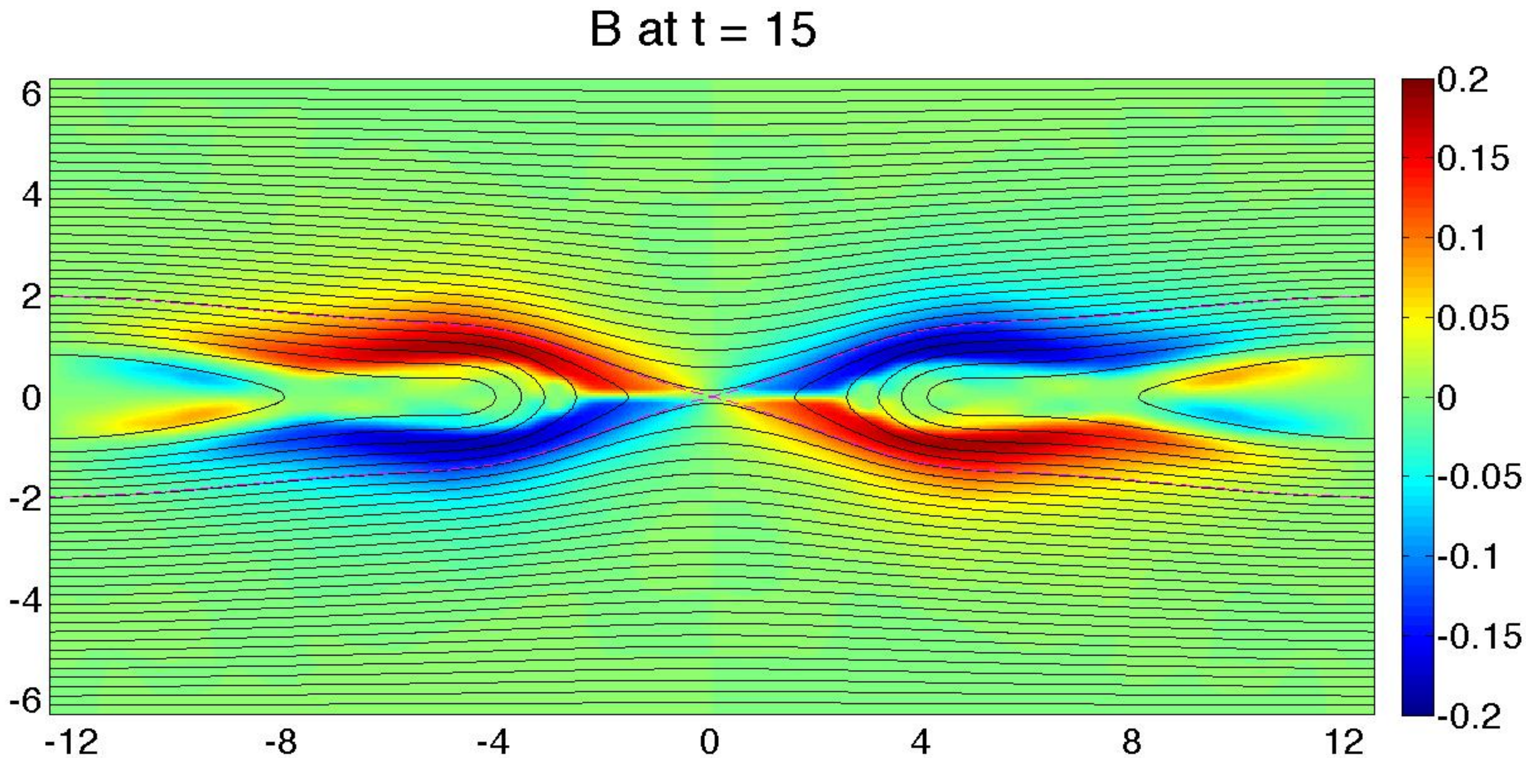
GEM magnetic reconnection challenge



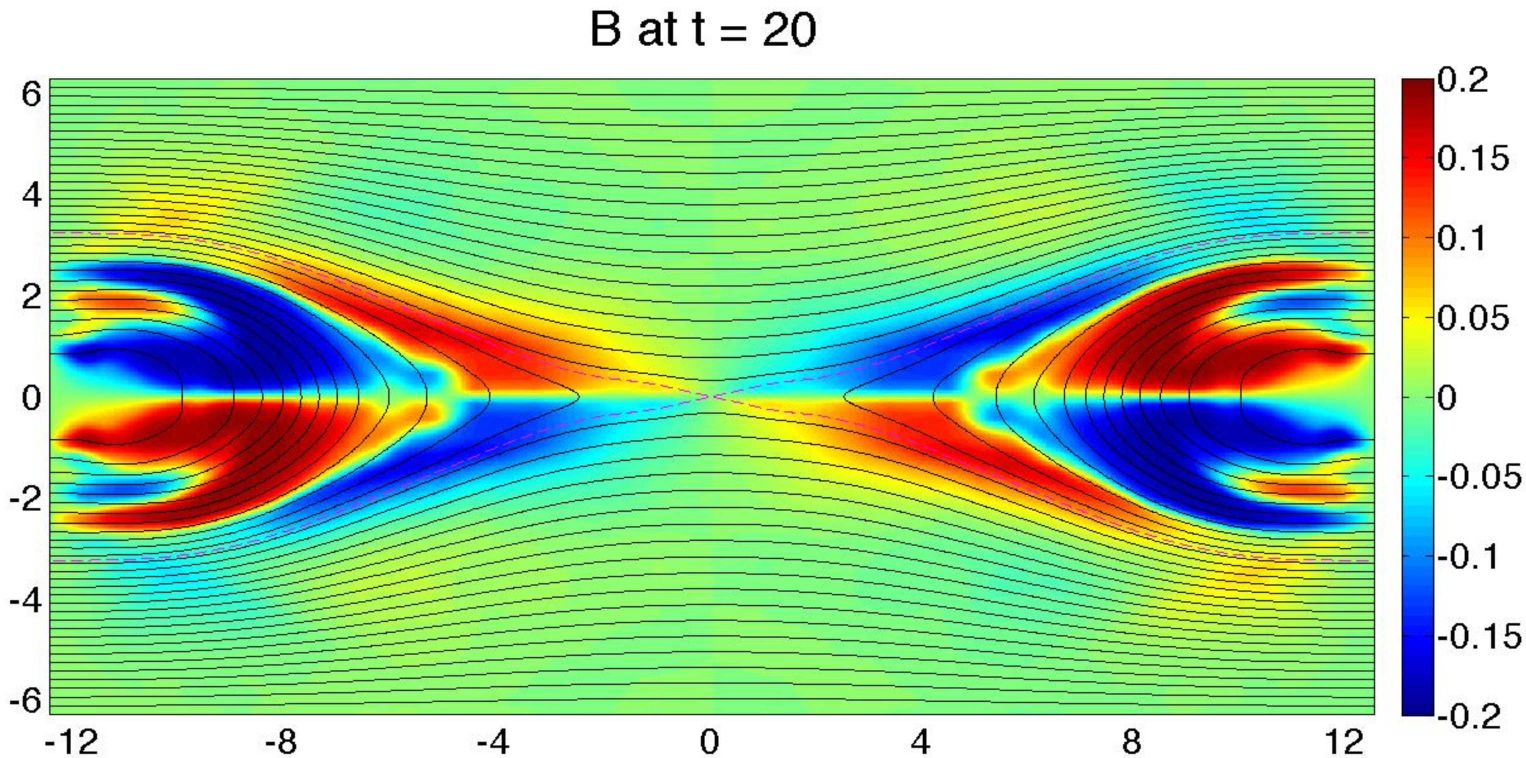
GEM magnetic reconnection challenge



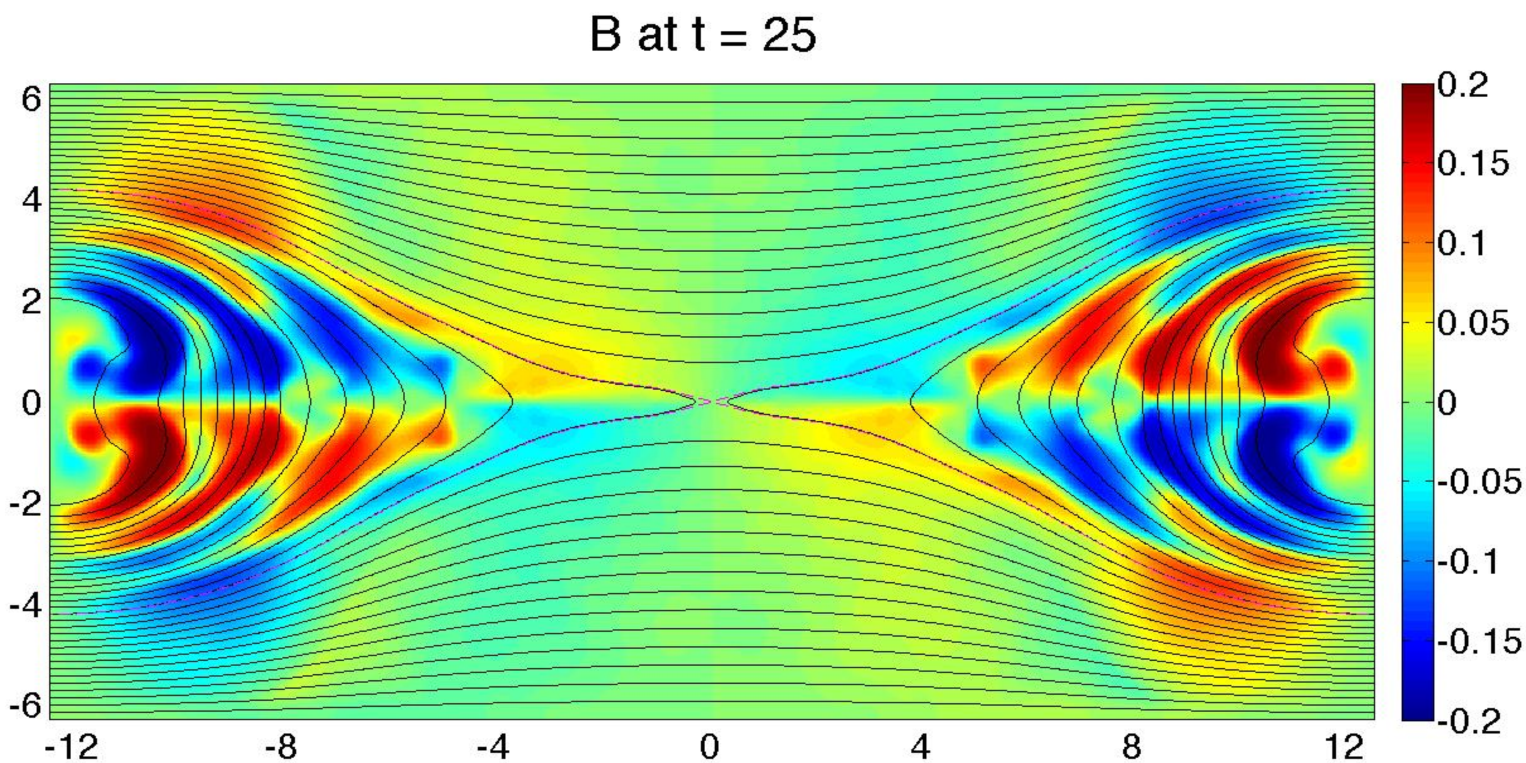
GEM magnetic reconnection challenge



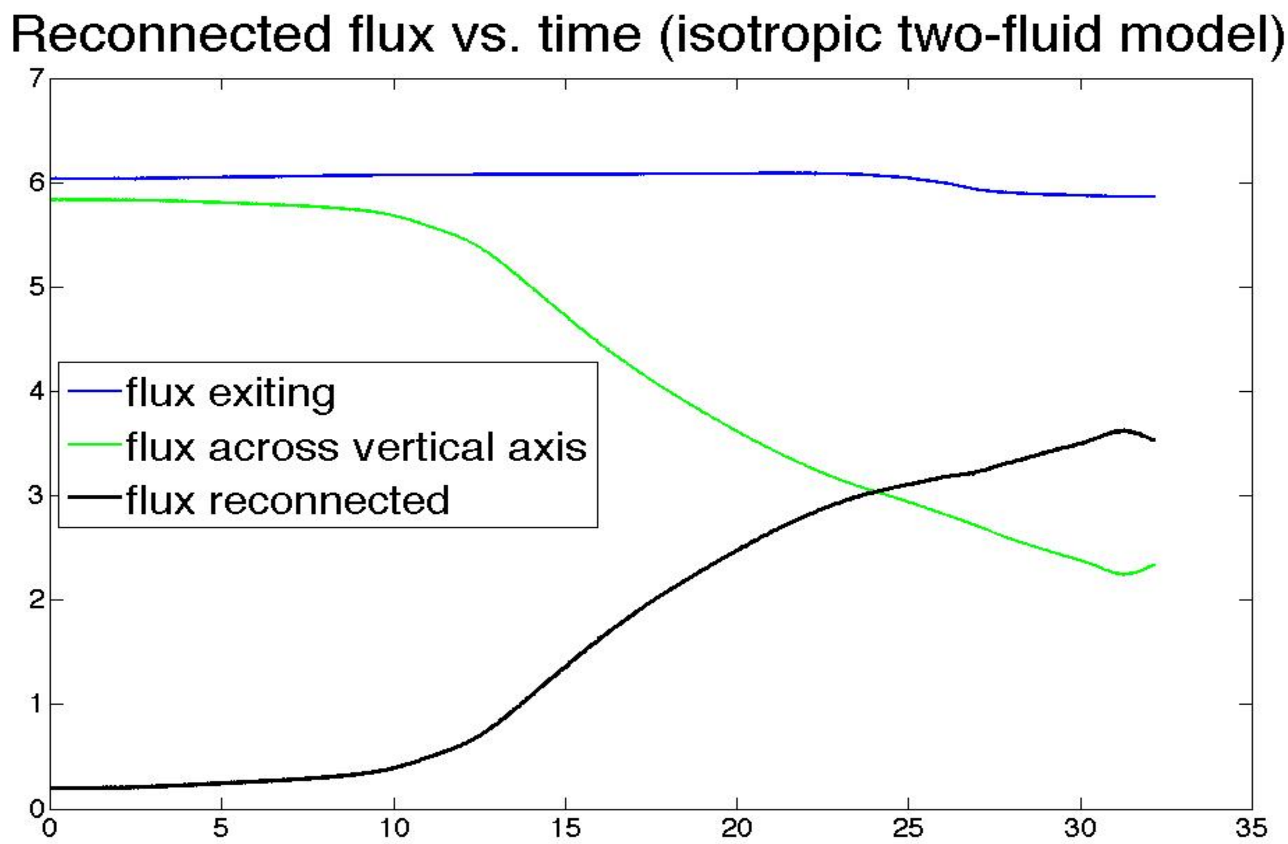
GEM magnetic reconnection challenge



GEM magnetic reconnection challenge



GEM magnetic reconnection challenge



($m_i/m_e = 25$, $T_i/T_e = 5$, mx=128, my=64)

Reconnection in the GEM context

Definition 1. *The reconnected flux* F_{recon} *is defined by*

$$F_{\mathrm{left}}(t) := \int_0^{y_{\max}} B_1\,dy, \qquad\qquad F_{\mathrm{recon}}(t) := F_{\mathrm{left}}(0) - F_{\mathrm{left}}(t).$$

Proposition 1. *The rate of reconnection is minus the value of the out-of-plane component of the electric field at the origin (i.e. the X-point).* [2]

Proof:

$$d_t F_{\mathrm{recon}}(t) = -d_t F_{\mathrm{left}}(t) \quad = -\int_0^{y_{\max}} \partial_t B_1\,dy = \int_0^{y_{\max}} \partial_y E_3\,dy \quad = -E_3(0),$$

since E_3 is zero at the conducting wall.

[2]This confirms the theoretical fact that an MHD model which only includes the $\mathbf{B}\times\mathbf{u}$ and Hall terms in Ohm's law cannot give fast reconnection, since by symmetries both these terms must vanish at the origin.

GEM: Ohm's law at the origin

Symmetries at the origin reduce Ohm's law to:

$$\begin{aligned}
\mathbf{E}_3 = & \eta J_3 && \text{(resistive term)} \\
& + \frac{1}{\rho}(\tilde{m}_e(\partial_{x_1}\mathbb{P}_{i,1,3} + \partial_{x_2}\mathbb{P}_{i,2,3}) - \tilde{m}_i(\partial_{x_1}\mathbb{P}_{e,1,3} + \partial_{x_2}\mathbb{P}_{e,2,3})) && \text{(pressure term)} \\
& + \frac{\tilde{m}_i\tilde{m}_e}{\rho}\Big(\partial_t\mathbf{J}_3 + J_3\nabla\cdot\mathbf{u} + u_3\nabla\cdot\mathbf{J} + \frac{\tilde{m}_e - \tilde{m}_i}{\rho}J_3\nabla\cdot\mathbf{J}\Big) && \text{(inertial term).}
\end{aligned}$$

One of these terms must be nonzero at the origin for reconnection to occur.

GEM: electric field at the origin

The momentum equation for the ions (solved for $\mathbf{E}$) is:

$$\mathbf{E} = \frac{-\mathbf{R}_i}{en_i} + \mathbf{B} \times \mathbf{u}_i + \frac{\nabla \cdot \mathbb{P}_i}{en_i} + \frac{m}{e}(\partial_t \mathbf{u}_i + \mathbf{u}_i \cdot \nabla \mathbf{u}_i),$$

where $\mathbf{R}_i$ is collisional force.

At the X-point this reduces to:

$$d_t F_{\mathrm{recon}}(t) = \mathbf{E}_3(0) = \left[\frac{-\mathbf{R}_i}{en_i} + \frac{\nabla \cdot \mathbb{P}_i}{en_i} + \frac{m}{e}\partial_t \mathbf{u}_i\right]_3,$$

where only the out-of-plane (third) component is nonzero.

In a perfectly collisionless, gyrotropic plasma, this reduces to

$$d_t F_{\mathrm{recon}}(t) = \mathbf{E}_3 = \frac{m}{e}\partial_t \mathbf{u}_{i3},$$

i.e., reconnected flux should exactly track with the current at the origin.

symmetric pair plasma

For the GEM problem, in the case of symmetry between positrons and electrons (equal temperatures) we get complete symmetry between species.

For resistive gyrotropic symmetric pair plasma,

$$\mathbf{E} = \alpha \mathbf{u}_i + (m/e)\partial_t(\mathbf{u}_i),$$

where the coefficient α specifies resistive drag force.

Plugging this into the Ampere-Maxwell law gives a harmonic oscillator (damped in case $\mathbf{R}_i \neq 0$) for u_{i3} forced by $\nabla \times \mathbf{B}$.

Integrating gives:

$$F_{\mathrm{recon}}(t) = -\int_0^t E_3(t) = (m/e)\mathbf{u}_i(0) - \mathbf{u}_i(t) - \alpha \int_0^t \mathbf{u}_i$$

Original studies of GEM problem

The original GEM challenge paper [3] studied the GEM problem using PIC, Hall MHD, and resistive MHD models and obtained fast reconnection in models which included the Hall effect.

They found that MHD with large anomalous (e.g. current-dependent) resistivity did not exhibit the quadrupole out-of-plane magnetic field pattern that appears to characterize models which incorporate the Hall term.

BIRN ET AL.: GEM RECONNECTION CHALLENGE

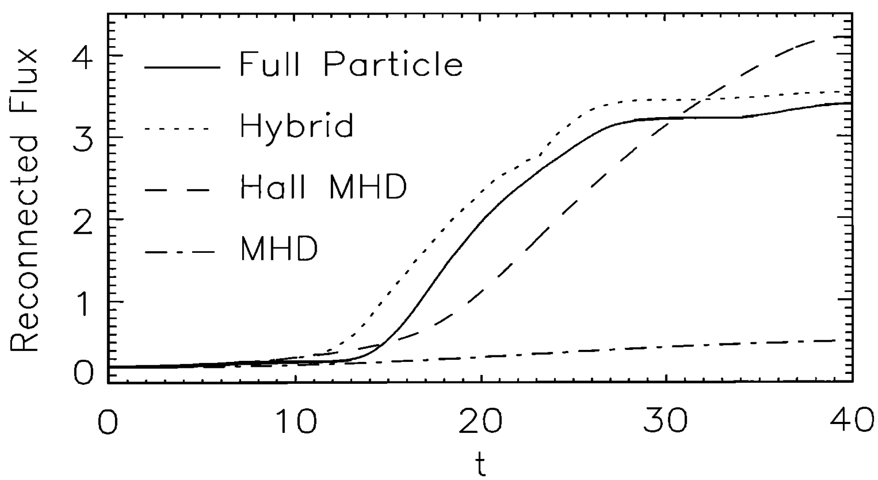


Figure 1. The reconnected magnetic flux versus time from a variety of simulation models: full particle, hybrid, Hall MHD, and MHD (for resistivity $\eta = 0.005$).

[3] J. Birn, J.F. Drake, M.A. Shay, B.N. Rogers, R.E. Denton, M. Hesse, M. Kuznetsova, Z.W. Ma, A. Bhattacharjee, A. Otto, and P.L. Pritchett. Geospace environmental modeling (GEM) magnetic reconnection challenge. *Journal of Geophysical Research – Space Physics*, 106:3715–3719, 2001.

PIC studies of pair plasma

The original GEM studies prompted the question:

Is the Hall term necessary for fast reconnection?

In **pair plasmas** (a.k.a. *electron-positron* plasmas), $m_i/m_e = 1$ and the Hall term is absent.

Bessho and Bhattacharjee[4] used particle-in-cell (PIC) simulations to study the following variations of the GEM problem:

problem	m_i/m_e	T_i/T_e	**fast reconnection?**	**quadrupole $\mathbf{B}_{\textbf{out}}$?**
original GEM	25	5	yes	quadrupole structure
(bridge)	1	5	yes	no quadrupole structure
symmetric pair plasma	1	1	yes	no $\mathbf{B}_{\text{out}}$

For equal temperatures they plotted the electric field and the terms of Ohm's law along a line through the vertical axis for equal temperature pair plasma and verified that at the X-point all terms vanish except the pressure term (as theoretically predicted).

[4] N. Bessho, A. Bhattacharjee. *Collisionless reconnection in an electron-positron plasma.* Physical Review Letters, **95**, 245001 (2005).

Our question

Bessho and Bhattacharjee's results prompted us to ask whether fast reconnection can occur in the absence of the Hall term *and* pressure anisotropy. That is,

Can the *inertial term alone* admit fast reconnection?

We therefore studied reconnection in an isotropic model of collisionless pair plasma:

We use the isotropic ideal two-fluid model:

① Euler gas-dynamics for the positive species
② Euler gas-dynamics for the negative species
③ Maxwell's equations for the electromagnetic field

There is no direct coupling between the two species.

Five-moment collisionless two-fluid model

The collisionless two-fluid equations we solved[5] were

$$\partial_t \begin{bmatrix} \rho_i \\ \rho_i \mathbf{u}_i \\ \mathcal{E}_i \\ \rho_e \\ \rho_e \mathbf{u}_e \\ \mathcal{E}_e \end{bmatrix} + \nabla \cdot \begin{bmatrix} \rho_i \mathbf{u}_i \\ \rho_i \mathbf{u}_i \mathbf{u}_i + p_i \mathbb{I} \\ \mathbf{u}_i \left(\mathcal{E}_i + p_i \right) \\ \rho_e \mathbf{u}_e \\ \rho_e \mathbf{u}_e \mathbf{u}_e + p_e \mathbb{I} \\ \mathbf{u}_e \left(\mathcal{E}_e + p_e \right) \end{bmatrix} = \begin{bmatrix} 0 \\ \sigma_i (\mathbf{E} + \mathbf{u}_i \times \mathbf{B}) \\ \sigma_i \mathbf{u}_i \cdot \mathbf{E} \\ 0 \\ \sigma_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) \\ \sigma_e \mathbf{u}_e \cdot \mathbf{E} \end{bmatrix},$$

$$\partial_t \begin{bmatrix} \mathbf{B} \\ \mathbf{E} \end{bmatrix} + \begin{bmatrix} \nabla \times \mathbf{E} + \chi \nabla \psi \\ -c^2 \nabla \times \mathbf{B} + \chi c^2 \nabla \phi \end{bmatrix} = \begin{bmatrix} 0 \\ -\mathbf{J}/\epsilon \end{bmatrix}, \partial_t \begin{bmatrix} \psi \\ \phi \end{bmatrix} + \begin{bmatrix} \chi c^2 \nabla \cdot \mathbf{B} \\ \chi \nabla \cdot \mathbf{E} \end{bmatrix} = \begin{bmatrix} 0 \\ \chi \sigma / \epsilon \end{bmatrix}.$$

The correction potentials ψ and ϕ are for numerical divergence cleaning purposes.

We used Discontinuous Galerkin, third order in space and time.

[5]These equations were studied extensively in

- U. Shumlak and J. Loverich. Approximate Riemann solver for the two-fluid plasma model. *J. Comp. Phys.*, 187:620–638, 2003, and
- A. Hakim, J. Loverich, and U. Shumlak, A high-resolution wave propagation scheme for ideal two-fluid plasma equations. *J. Comp. Phys.*, 219:418–442, 2006.

GEM studies

We ran our model for Bessho and Bhattacharjee's settings for the GEM problem. The reconnection region was long and narrow (i.e. a high aspect ratio, as in Sweet-Parker reconnection), triggering the tearing mode instability. This made it difficult to demonstrate convergence.

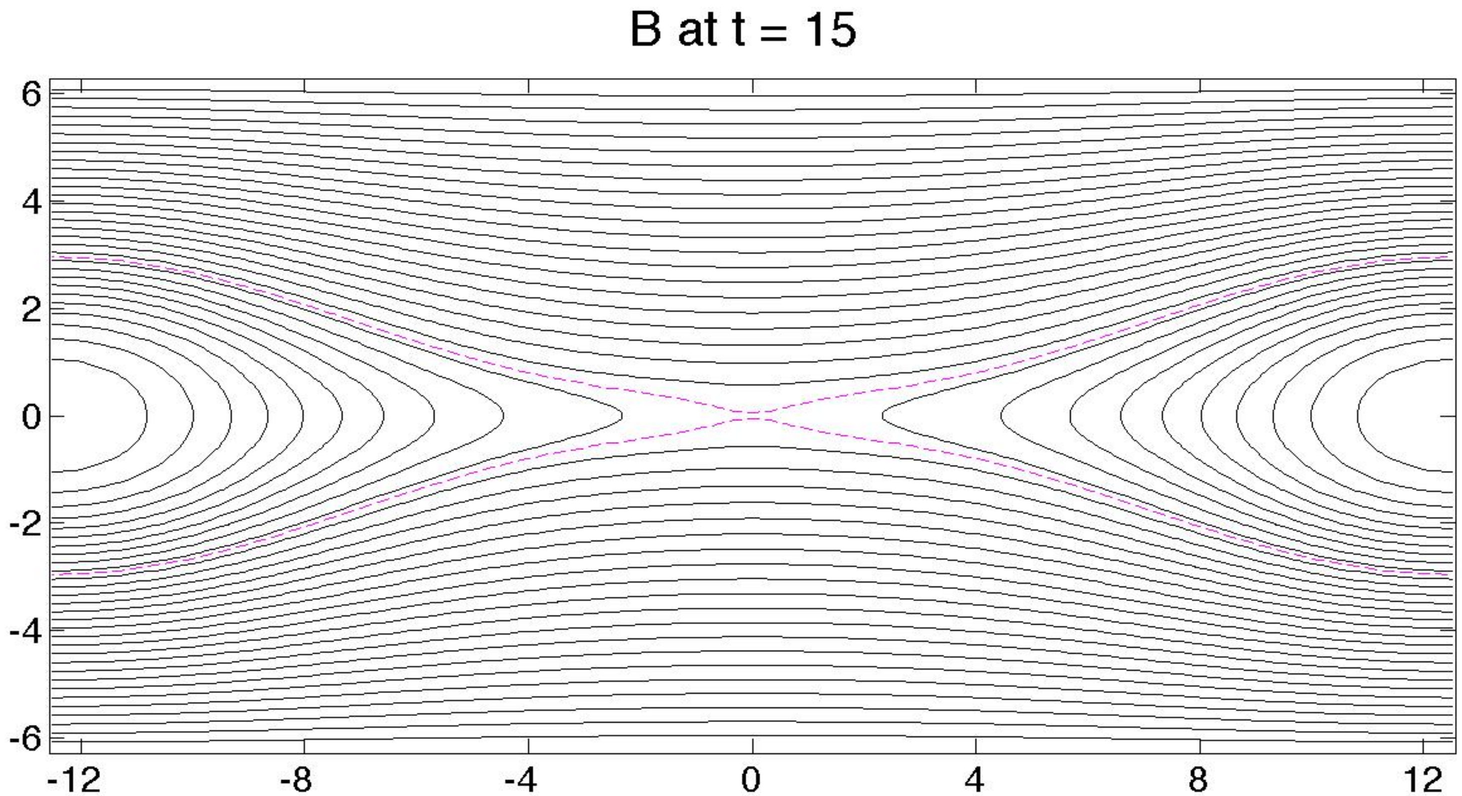
Recent PIC simulations have studied the aspect ratio of fast reconnection. [6]

We instead wished to focus on the rate of reconnection/tearing, so we halved the size of the domain.

[6] M. Swisdak, Yi-Hsin Liu, J.F. Drake (2008) Development of a turbulent outflow during electron-positron magnetic reconnection. *The Astrophysical Journal*, 680:999–1008.

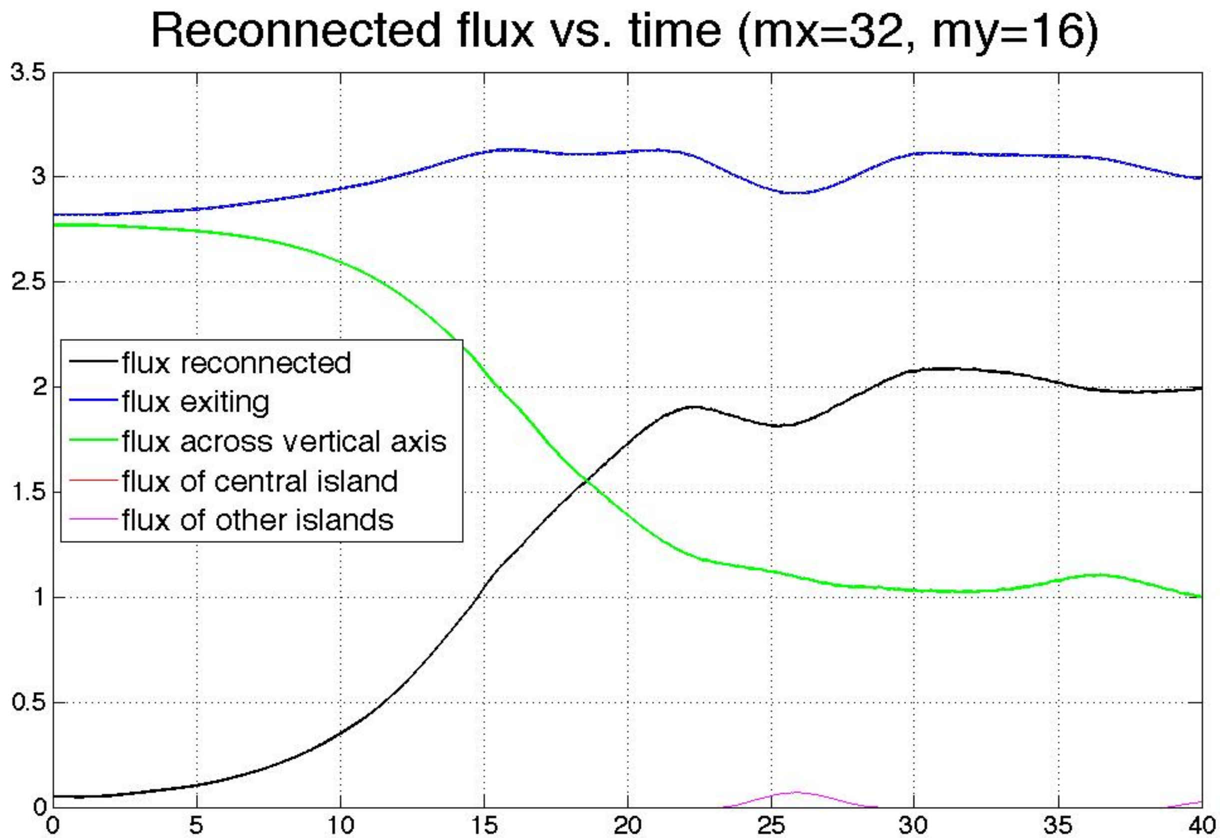
Results

Our simulations of isotropic pair plasma show fast reconnection and appear to be converged.



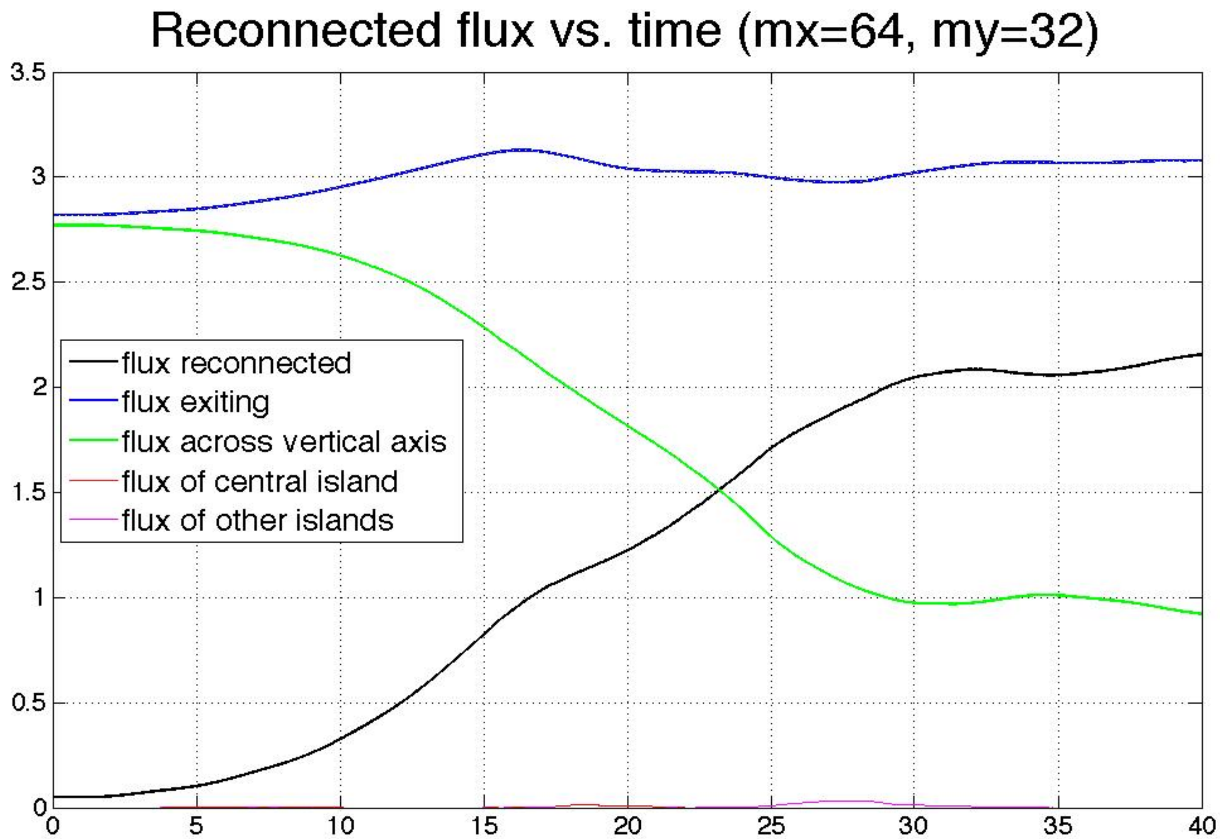
Reconnected field lines in isotropic symmetric pair plasma.

Isotropic pair plasma, $T_i/T_e = 1$



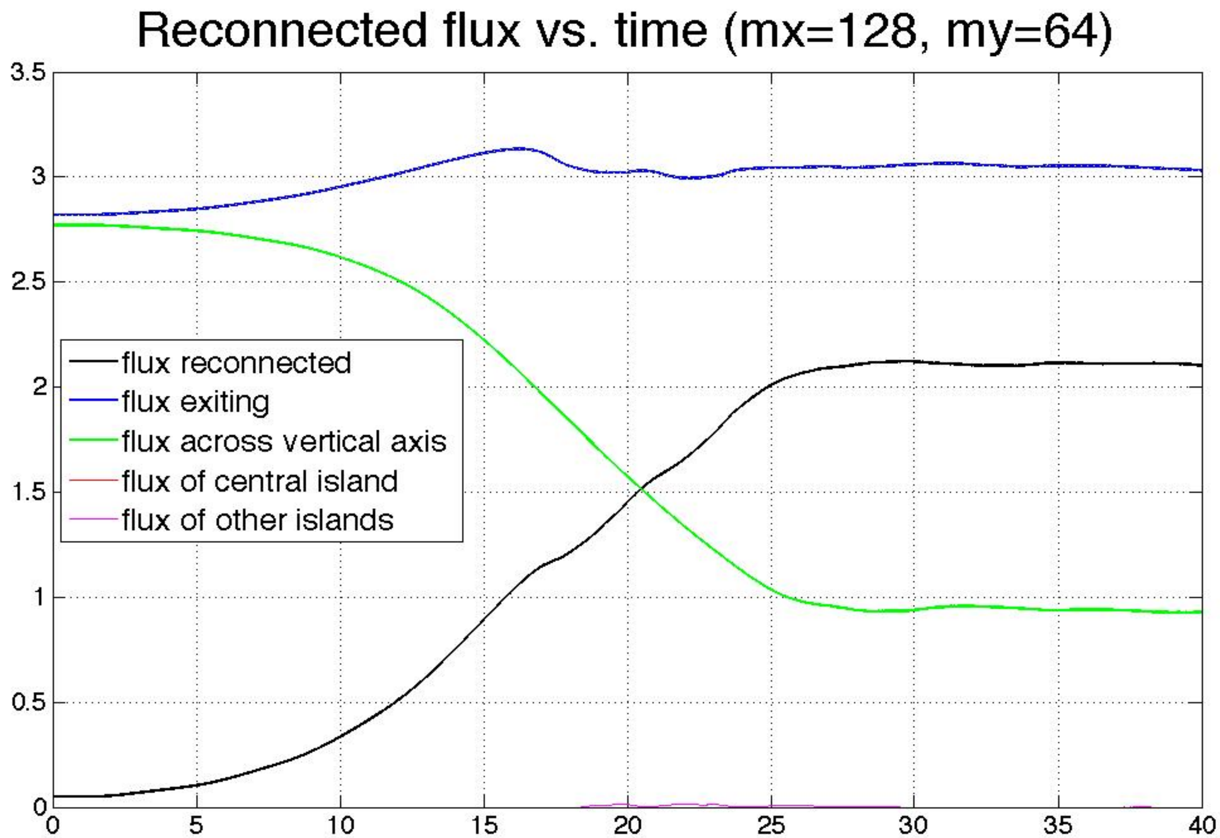
Reconnection in isotropic symmetric pair plasma (coarse mesh).

Isotropic pair plasma, $T_i/T_e = 1$



Reconnection in isotropic symmetric pair plasma (medium mesh).

Isotropic pair plasma, $T_i/T_e = 1$



Reconnection in isotropic symmetric pair plasma (fine mesh).

Ten-moment studies

We have also studied the GEM problem with a ten-moment (anisotropic pressure) model. The ten-moment equations assume that the generalized heat flux $\rho\langle\mathbf{ccc}\rangle$ is zero, i.e., that the pressure tensor is an anisotropic Gaussian.

We find that:

① With no collisions reconnection is slow and saturates at a low level.

② Relaxation toward an isotropic pressure tensor admits fast reconnection.

Ten-moment equations

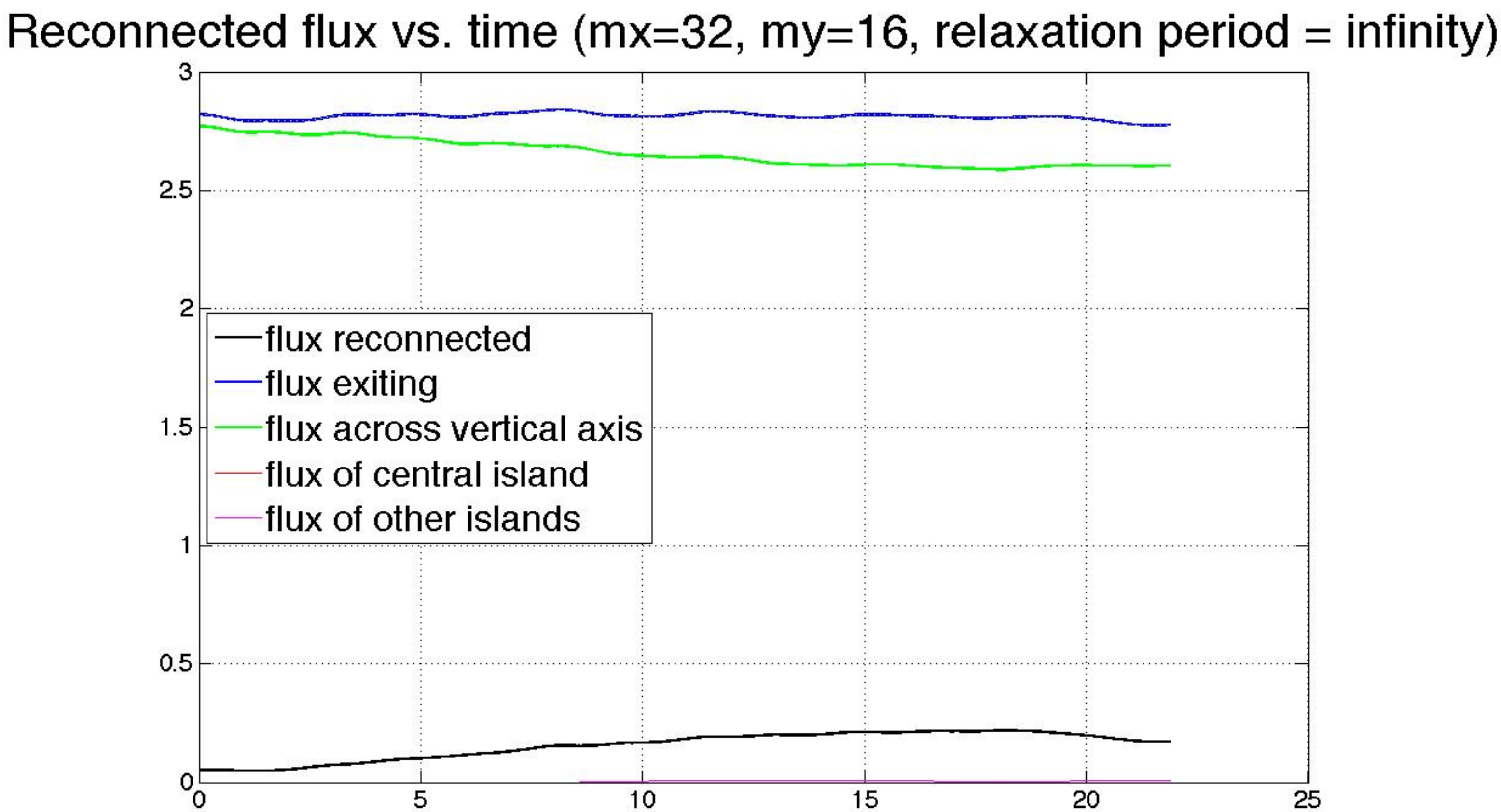
The ten-moment model replaces the gas-dynamic energy $\mathcal{E}_s := \rho_s \langle v_s^2 \rangle / 2$ with an energy tensor $\mathbb{E}_s := \rho_s \langle \mathbf{v}_s \mathbf{v}_s \rangle$. ($\mathbf{v}_s$ is particle velocity and angle brackets denote statistical average over a small test volume). The nondimensionalized system of equations used in the collisionless ten-moment two-fluid model is

$$\partial_t \begin{bmatrix} \rho_i \\ \rho_e \\ \rho_i \mathbf{u}_i \\ \rho_e \mathbf{u}_e \\ \mathbb{E}_i \\ \mathbb{E}_e \end{bmatrix} + \nabla \cdot \begin{bmatrix} \rho_i \mathbf{u}_i \\ \rho_e \mathbf{u}_e \\ \mathbb{E}_i \\ \mathbb{E}_e \\ 3\,\mathrm{Sym}(\mathbf{u}_i \mathbb{E}_i) - 2\rho_i \mathbf{u}_i \mathbf{u}_i \mathbf{u}_i \\ 3\,\mathrm{Sym}(\mathbf{u}_e \mathbb{E}_e) - 2\rho_e \mathbf{u}_e \mathbf{u}_e \mathbf{u}_e \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \sigma_i \mathbf{E} + \mathbf{J}_i \times \mathbf{B} \\ \sigma_e \mathbf{E} + \mathbf{J}_e \times \mathbf{B} \\ 2\,\mathrm{Sym}(\mathbf{J}_i \mathbf{E} + \frac{q_i}{m_i} \mathbb{E}_i \times \mathbf{B}) \\ 2\,\mathrm{Sym}(\mathbf{J}_e \mathbf{E} + \frac{q_e}{m_e} \mathbb{E}_e \times \mathbf{B}) \end{bmatrix},$$

$$\partial_t \begin{bmatrix} c\mathbf{B} \\ \mathbf{E} \end{bmatrix} + c\nabla \times \begin{bmatrix} \mathbf{E} \\ -c\mathbf{B} \end{bmatrix} = \begin{bmatrix} 0 \\ -\mathbf{J}/\epsilon \end{bmatrix}, \text{ and } \nabla \cdot \begin{bmatrix} c\mathbf{B} \\ \mathbf{E} \end{bmatrix} = \begin{bmatrix} 0 \\ \sigma/\epsilon \end{bmatrix}.$$

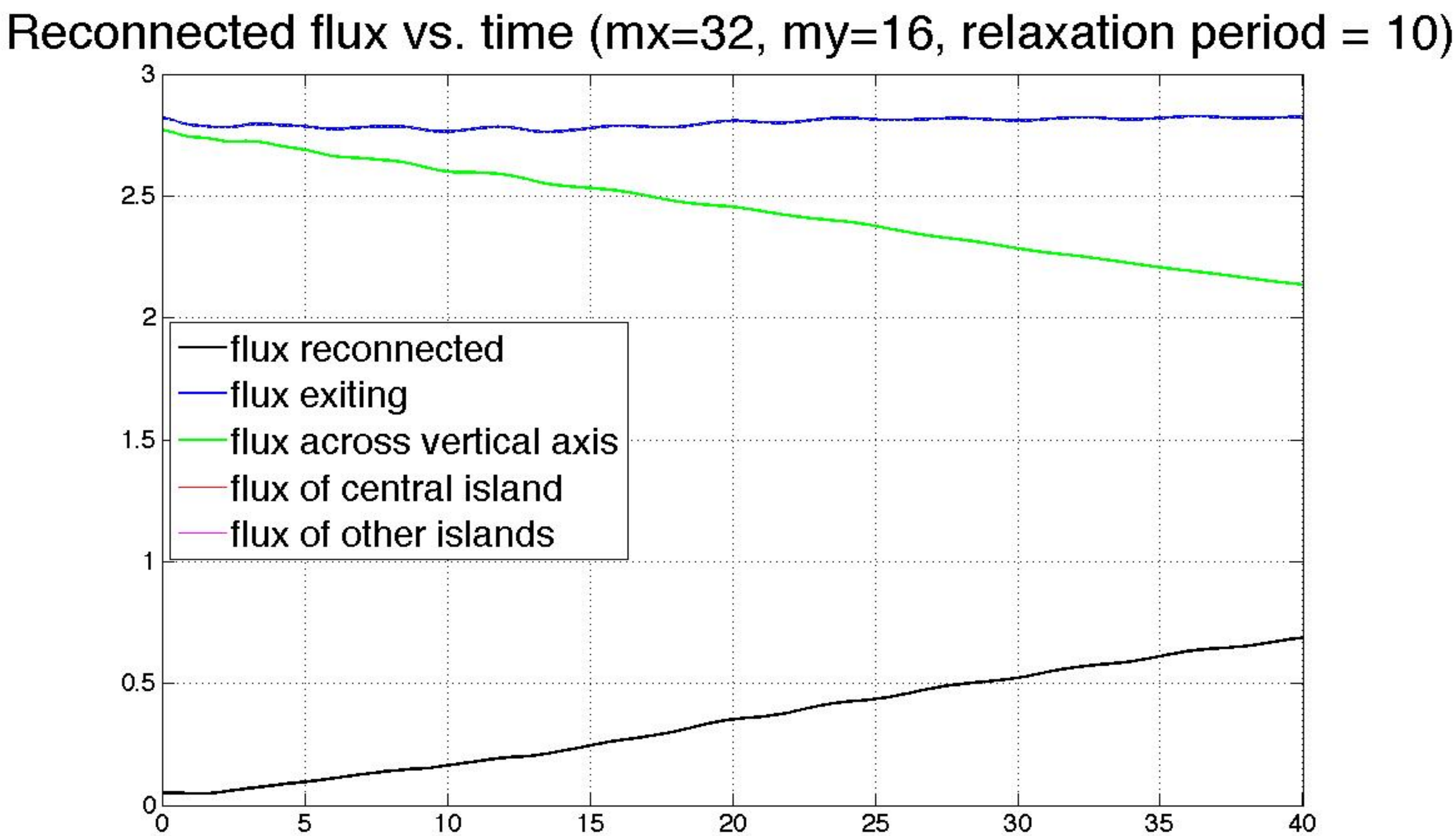
Here Sym denotes the symmetric part of its argument tensor (obtained by averaging over all permutations of subscripts).

Anisotropic results



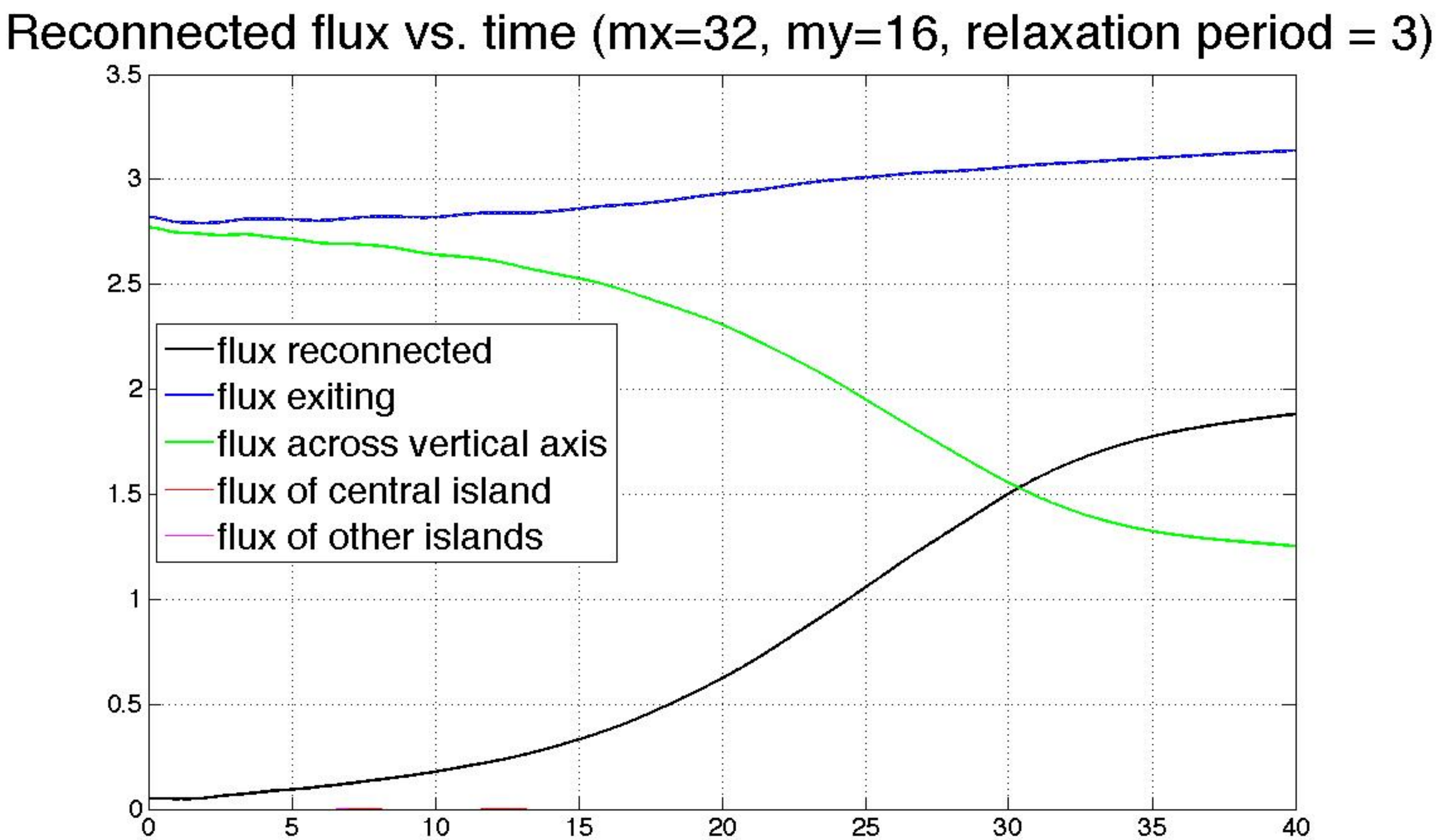
(Ten-moment pair plasma without relaxation toward isotropy).

Anisotropic results



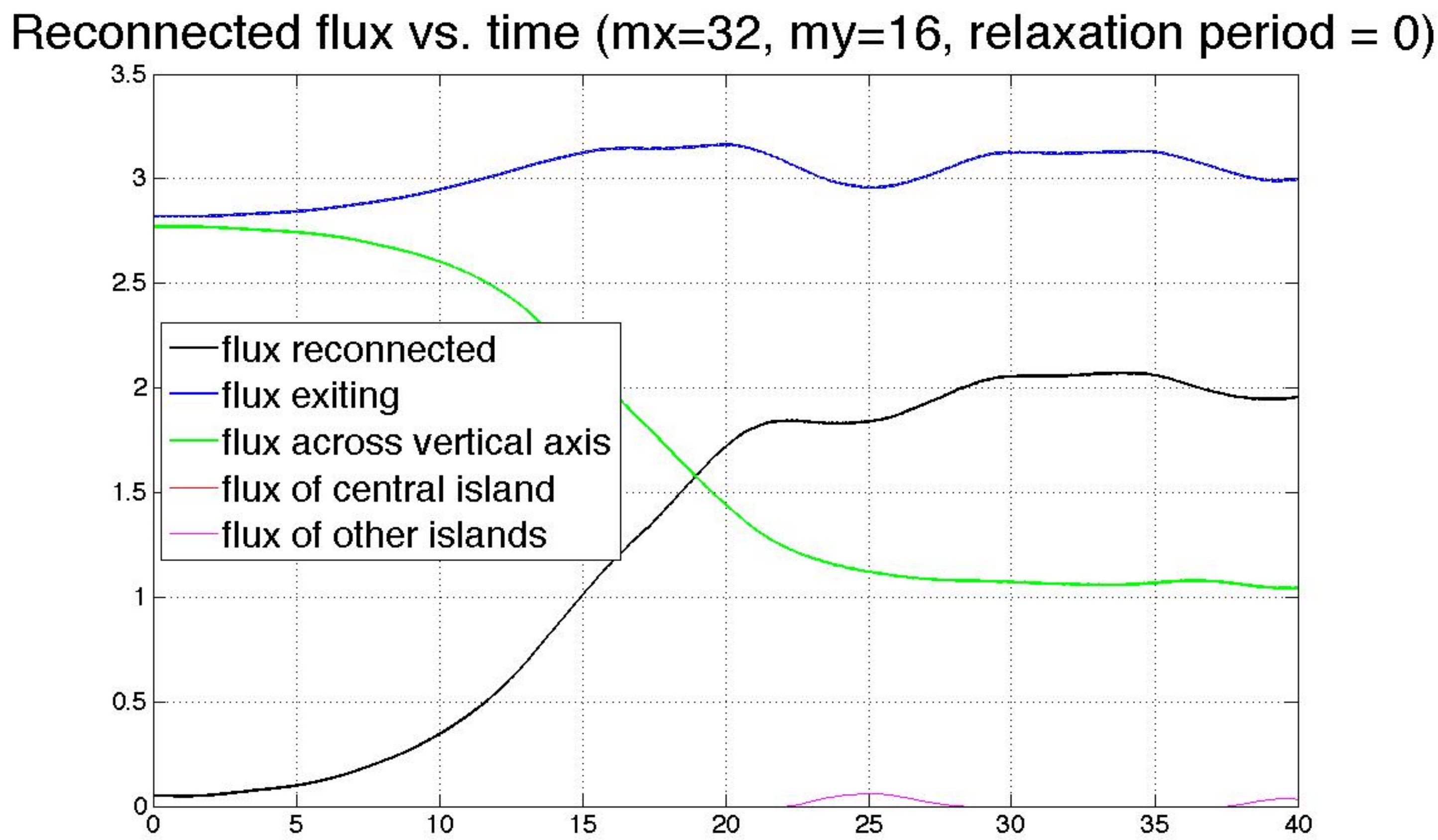
(Ten-moment pair plasma with slow relaxation toward isotropy).

Anisotropic results



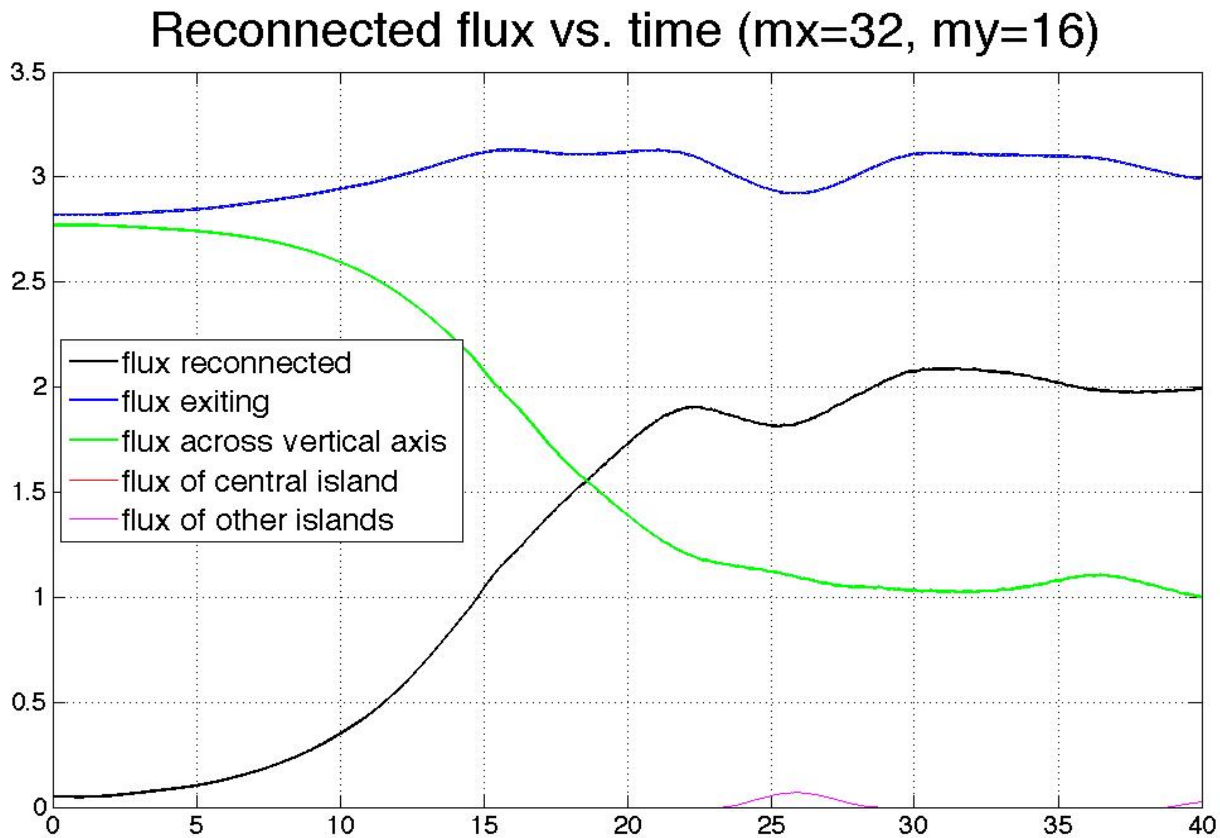
(Ten-moment pair plasma with fast relaxation toward isotropy).

Anisotropic results



(Ten-moment pair plasma with instantaneous relaxation toward isotropy).

(Recall: isotropic pair plasma, $T_i/T_e = 1$)



Reconnection in isotropic symmetric pair plasma (coarse mesh).

Further investigation

① How closely does the current at the origin track the reconnected flux as we refine the mesh?

② **Why does isotropization provide for reconnection?**

③ Why do we not get reconnection for the ten-moment model unless we isotropize?

④ **Can we incorporate generalized heat flux in the ten-moment model to provide for reconnection?**

⑤ How do resistivity and viscosity affect the rate of reconnection?

⑥ **Can we get fast reconnection for a fluid model of collisionless pair plasma with structure that agrees with PIC simulations?**

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