Space Weather - Homework - Due Dec 7

The purpose of this assignment is to make sure that you are prepared for the lecture on Thursday, December 7th.

This is a short and easy assignment, so you should be able to complete it before class. Please feel free to contact me if you have any questions. I will do a better job of teaching on Thursday if you bring questions to me. My contact information is:

E. Alec Johnson,
Departement Wiskunde
office: room 04.33,
Celestijnenlaan 200B
desk phone: +32 16 32 79 64
email: `Alec.Johnson@wis.kuleuven.be`
office hours: 10-12 and 14-17pm daily.

Problem 1

Derive the equations of the two-fluid Maxwell model by taking moments of the kinetic (i.e. "Vlasov") equation. You may neglect the deviatoric pressure, the heat flux, and all moments of the collision operator; *except resistive drag force in the momentum evolution equation.* Do not just plug e.g. $\chi = \mathbf{v}$ into the generic moment formula derived in the slides. Do a full derivation. You will need to use integration by parts to handle the velocity divergence.

We started this in class. For example:

$$\int_{\mathbf{v}} \mathbf{v}\Big(\partial_t f + \nabla_{\mathbf{x}} \cdot (\mathbf{v} f) + \nabla_{\mathbf{v}} \cdot (\mathbf{a} f) = \mathcal{C}\Big)$$

The term with the spatial derivative gives rise to nonlinearity; decompose the velocity as $\mathbf{v} = \mathbf{u} + \mathbf{c}$, where $\mathbf{u} := \langle \mathbf{v} \rangle$ is the fluid velocity and $\mathbf{c}$ is the *thermal velocity.* $\int_{\mathbf{v}} \mathbf{v} \nabla_{\mathbf{x}} \cdot (\mathbf{v} f) = \nabla_{\mathbf{x}} \cdot \int_{\mathbf{v}} \mathbf{vv} f$, and $\int_{\mathbf{v}} \mathbf{vv} f = \rho\langle \mathbf{vv} \rangle = \rho\langle (\mathbf{u}+\mathbf{c})(\mathbf{u}+\mathbf{c})\rangle = \rho\left(\mathbf{uu} + \mathbf{u}\cancel{\langle\mathbf{c}\rangle} + \cancel{\langle\mathbf{c}\rangle}\mathbf{u} + \langle\mathbf{cc}\rangle\right) = \rho\mathbf{uu} + \rho\langle\mathbf{cc}\rangle$; $\mathbb{P} := \rho\langle\mathbf{cc}\rangle$ contains unknown "bad stuff" arising from nonlinearity (for which a closure assumption is needed). Using integration by parts and assuming that f vanishes at $\mathbf{v} = \infty$, $\int_{\mathbf{v}} \mathbf{v} \nabla_{\mathbf{v}} \cdot (\mathbf{a} f) = \cancel{\int_{\mathbf{v}} \nabla_{\mathbf{v}} \cdot (\mathbf{av} f)} - \int_{\mathbf{v}} \mathbf{a} \cdot (\nabla_{\mathbf{v}} \mathbf{v}) f = -\int_{\mathbf{v}} \mathbf{a} \cdot \mathbb{I} f = -\int_{\mathbf{v}} \mathbf{a} f = -\rho\langle\mathbf{a}\rangle$. For the collision term you will need to make use of its conservation properties, e.g.: $\int_{\mathbf{v}} \mathbf{v}\mathcal{C} = \int_{\mathbf{v}} (\mathbf{u}+\mathbf{c})\mathcal{C} = \mathbf{u}\cancelto{0}{\int_{\mathbf{v}} \mathcal{C}} + \int_{\mathbf{v}} \mathbf{c}\mathcal{C} =: \mathbf{R}$. Finish this up and move on to energy evolution.

Problem 2

Integrate the equations of mass, momentum, and energy evolution over a fixed or convected control volume Ω and physically interpret each term. For example, choosing a fixed control volume Ω and integrating the momentum equation yields:

$$\left(\int_{\Omega} \rho\mathbf{u}\right)_t + \oint_{\partial\Omega} \widehat{\mathbf{n}} \cdot (\rho\mathbf{uu}) + \oint_{\partial\Omega} \widehat{\mathbf{n}} \cdot \mathbb{P} = \int_{\Omega} \sigma\mathbf{E} + \cdots,$$

which says that the rate of change of fluid momentum in Ω is minus the flow of momentum across the boundary minus the total pressure on the boundary plus the force of the electric field plus. . . .

Problem 3

Add the mass density evolution equations for ions and electrons to obtain a bulk density evolution equation. Do the same for bulk momentum. Handle the nonlinear term $\sum_{\mathrm{s}} \rho_{\mathrm{s}} \mathbf{u}_{\mathrm{s}} \mathbf{u}_{\mathrm{s}}$ by writing $\mathbf{u}_{\mathrm{s}} = \mathbf{u} + \mathbf{w}_{\mathrm{s}}$, where $\mathbf{u}$ is bulk fluid velocity. Separate out the "bad stuff" involving $\mathbf{w}_{\mathrm{s}}$, just like in Problem 1. You can call it "drift pressure."

Problem 4

Read the notes posted at the top of "`http://www.danlj.org/eaj/math/research/presentations/`". **Identify the first point in the notes that you do not understand and formulate a question.** *I will poll you for questions on Thursday!*