Simulation of Fast Magnetic Reconnection with a Ten-Moment Two-Fluid Model

E. Alec Johnson
Department of Mathematics, UW-Madison
Presented on June 16, 2010,
Hyp2010, Beijing.

Outline

① magnetic reconnection and the GEM problem

② plasma models

(a) kinetic (Vlasov, PIC)
(b) two-fluid (5- and 10-moment)
(c) one-fluid (MHD)

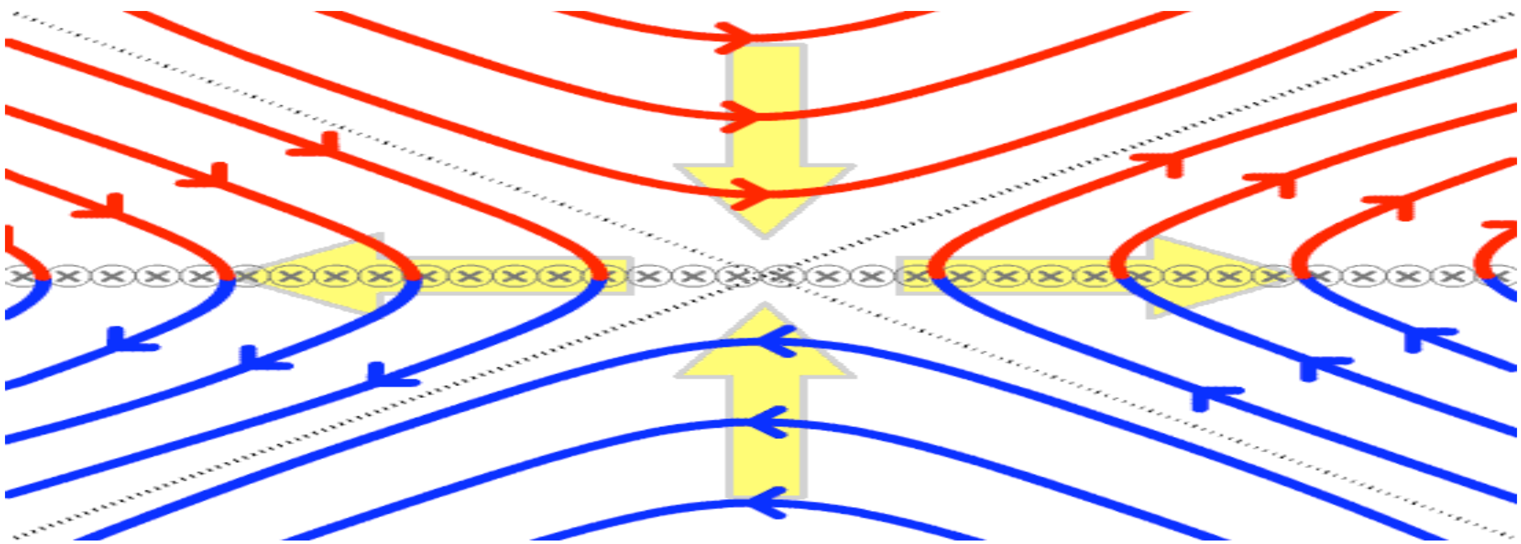
③ comparison of two-fluid simulations with kinetic simulations

Claim: The 10-moment two-fluid model is able to resolve the structure of the reconnection region fairly well.

Magnetic Reconnection

Plasma is a gas of charged particles. Charged particles gyrate around magnetic field lines. So magnetic field lines (like vortex lines) are approximately frozen into the plasma in the "ideal MHD" regime of small gyroradius and absence of collisional resistive drag.

The frozen-in-flux condition is violated near magnetic reconnection points. Specifically, adjacent strongly antiparallel magnetic field lines occasion strong magnetic field gradients and a large gyroradius, allowing magnetic field lines to diffuse and alter the topology of the magnetic field.



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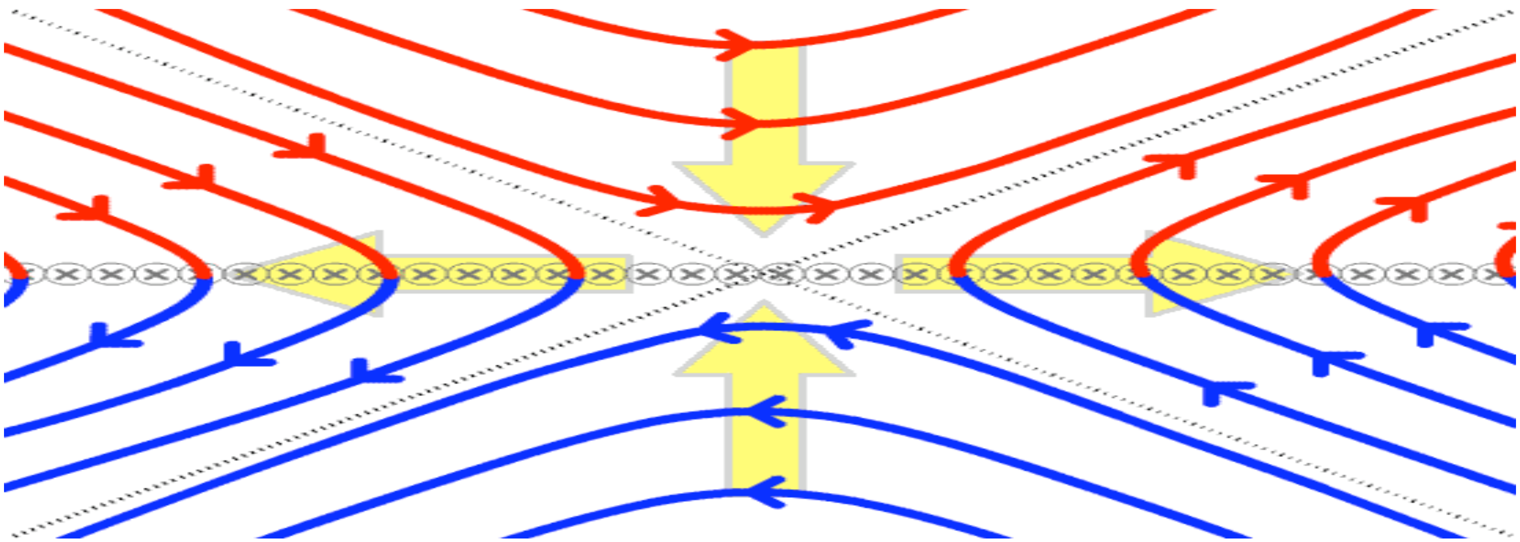

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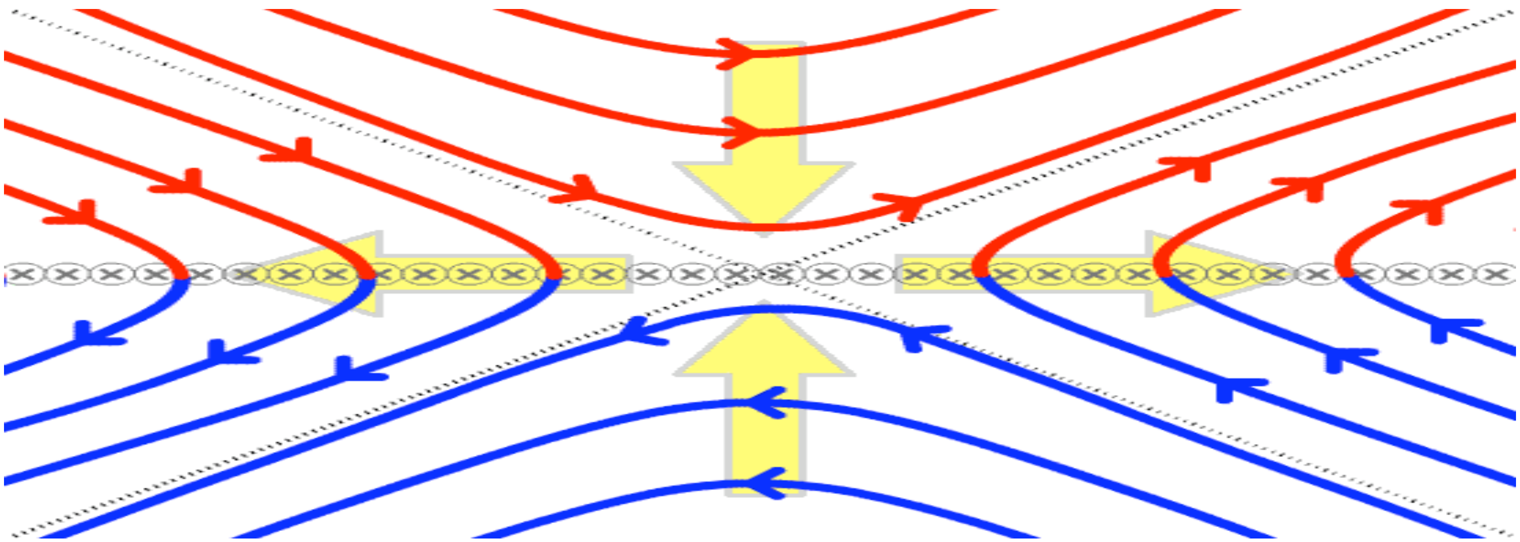

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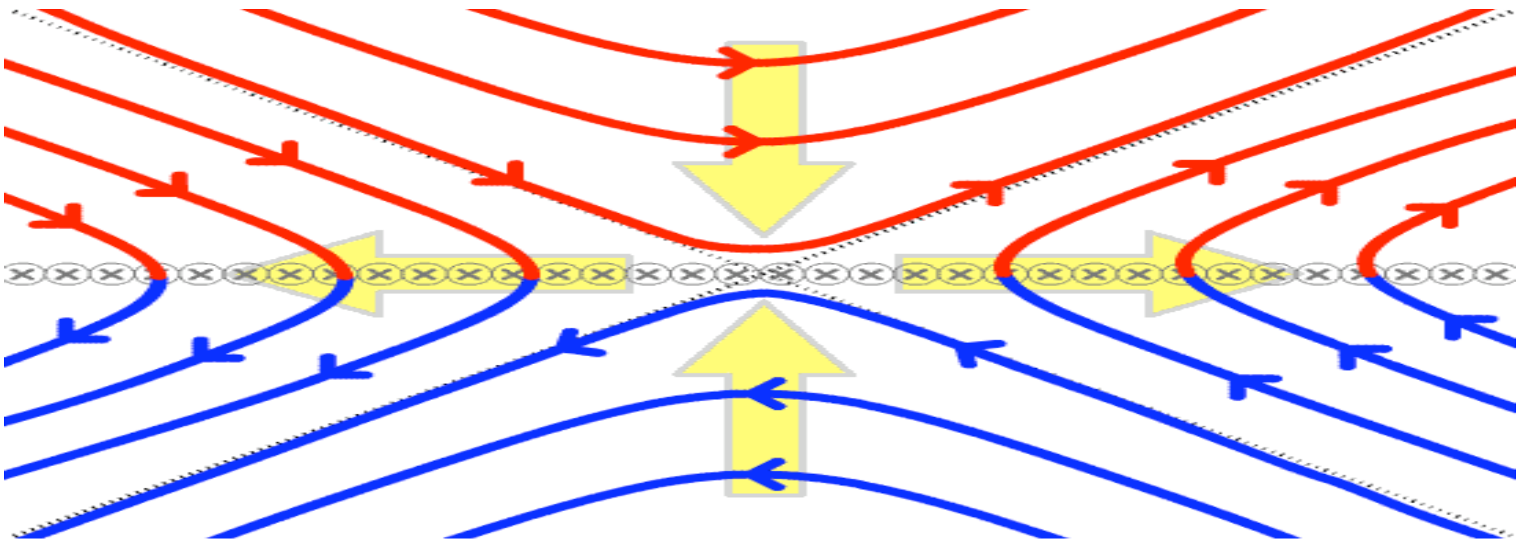

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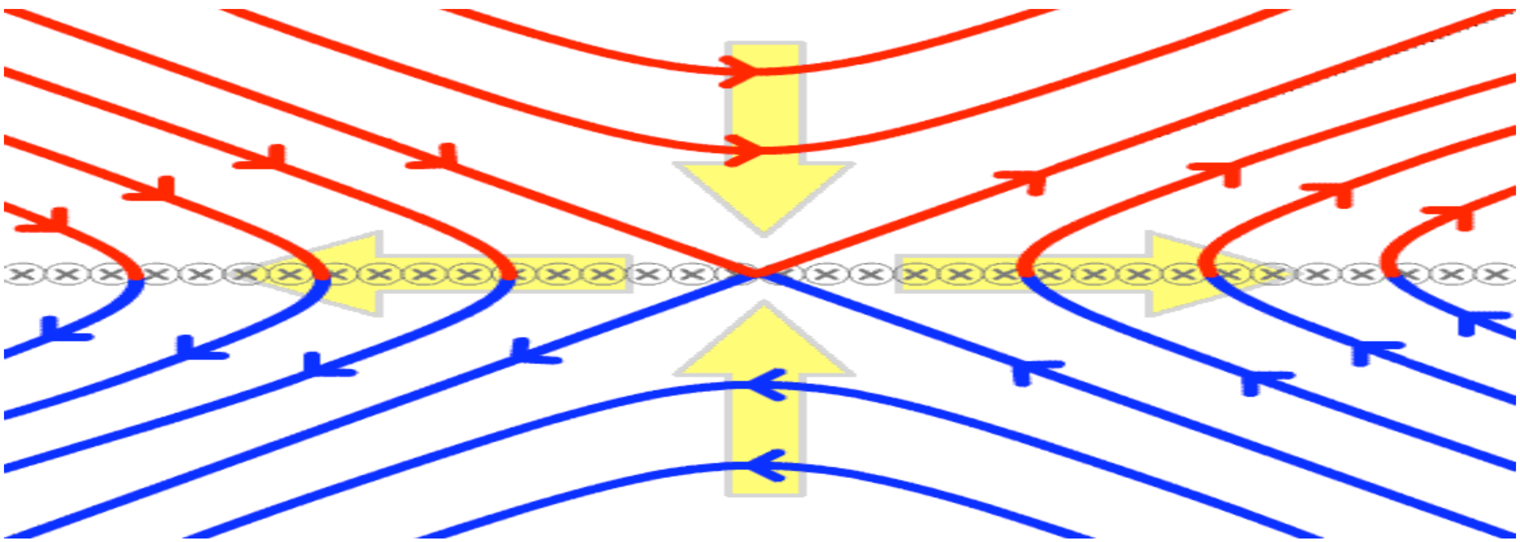

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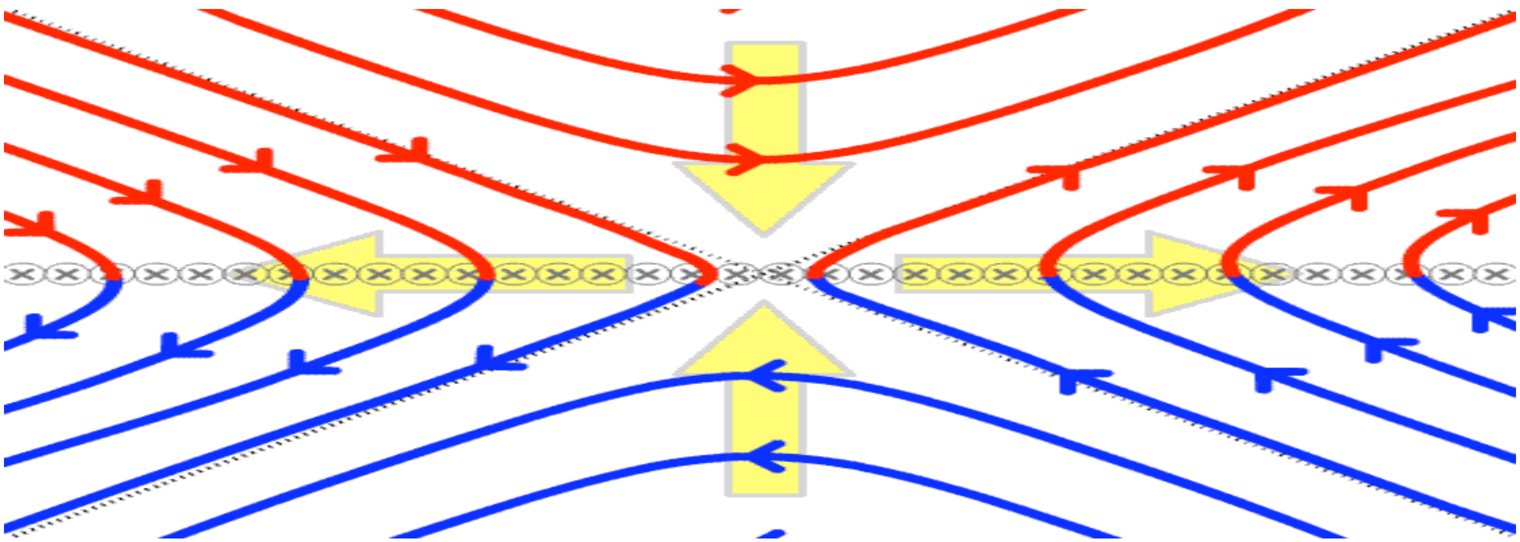

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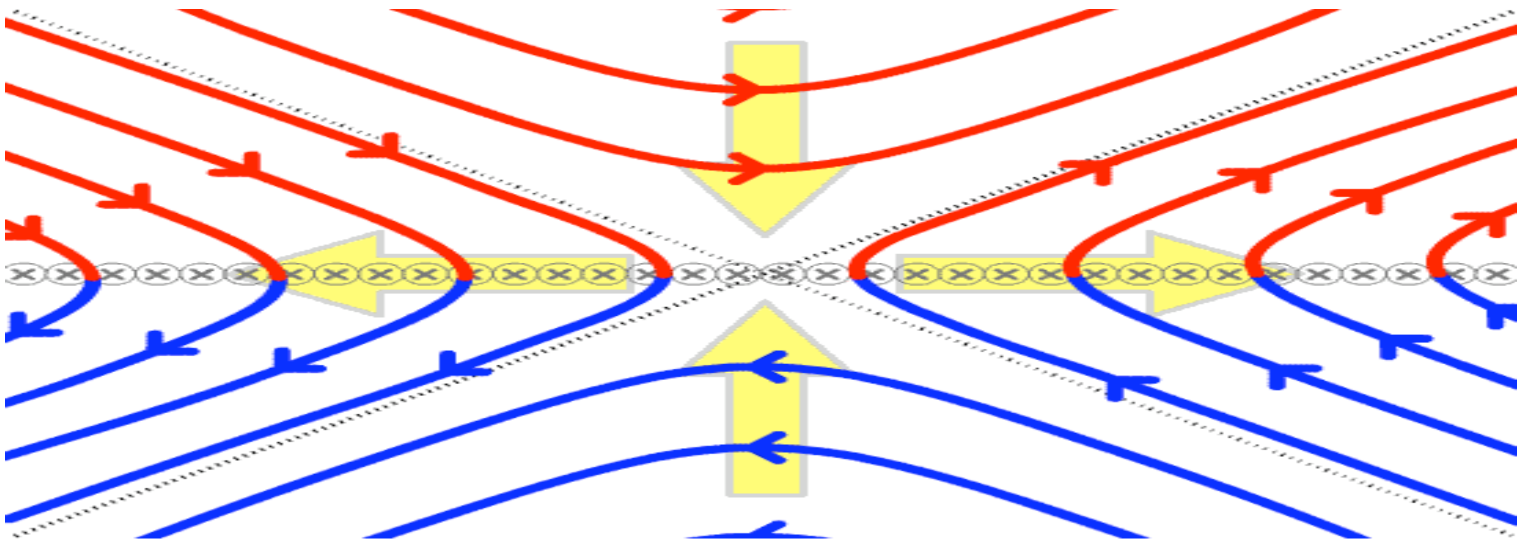

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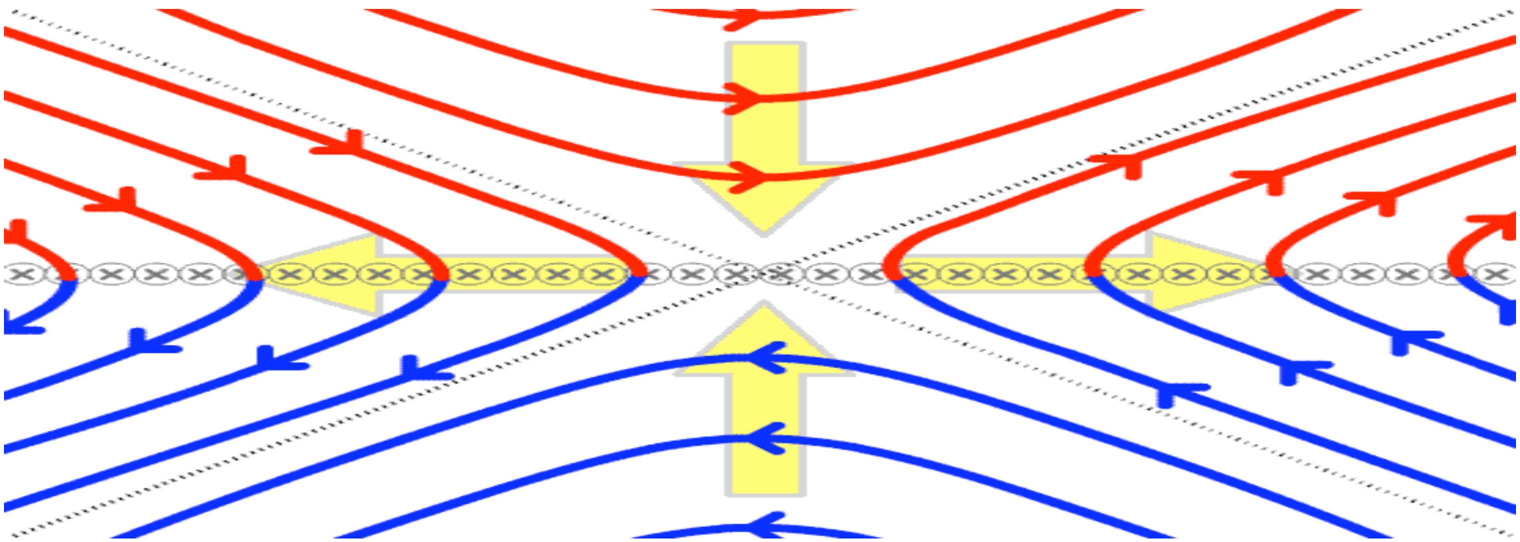

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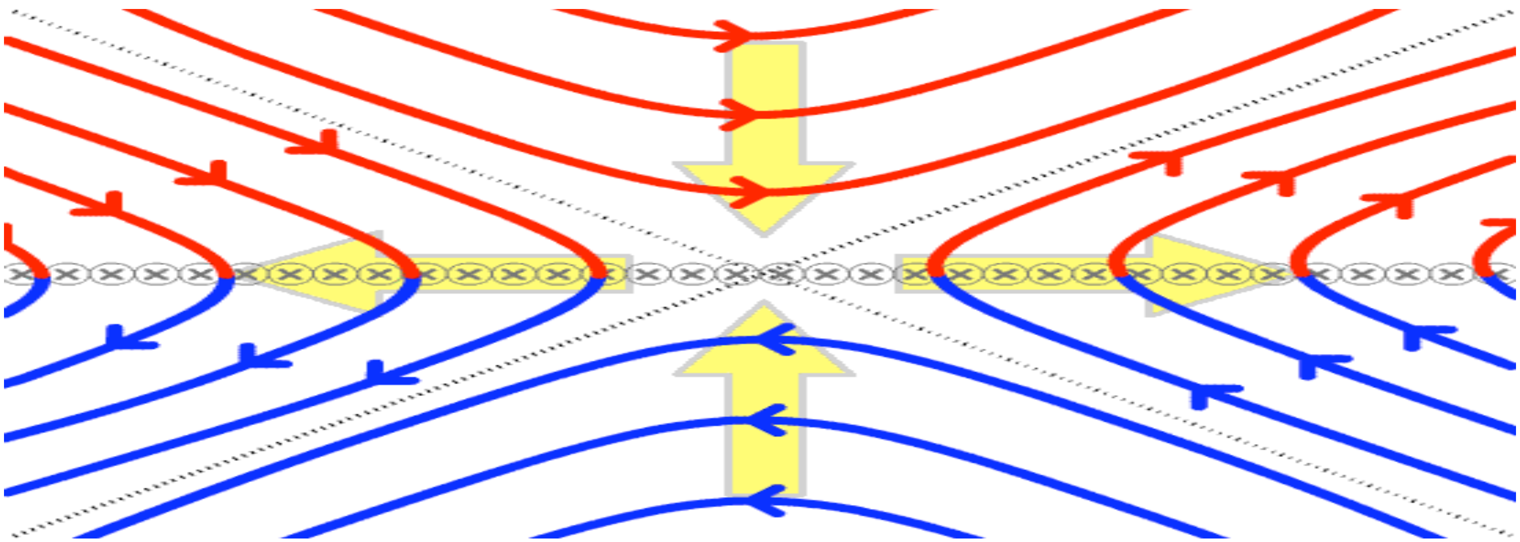
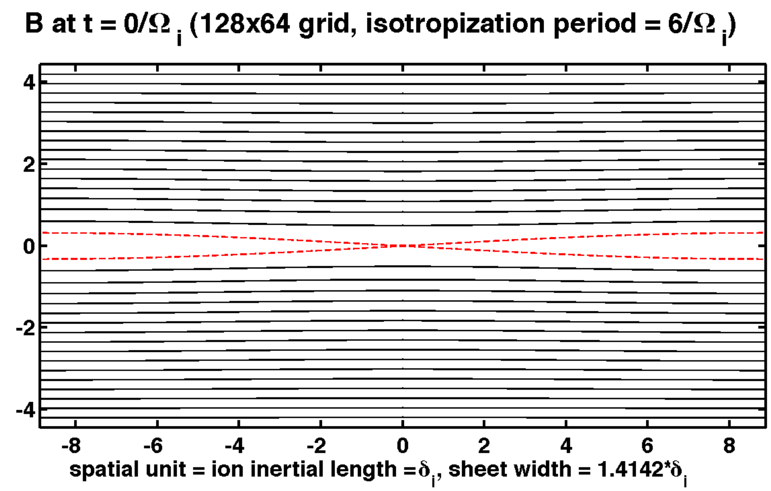
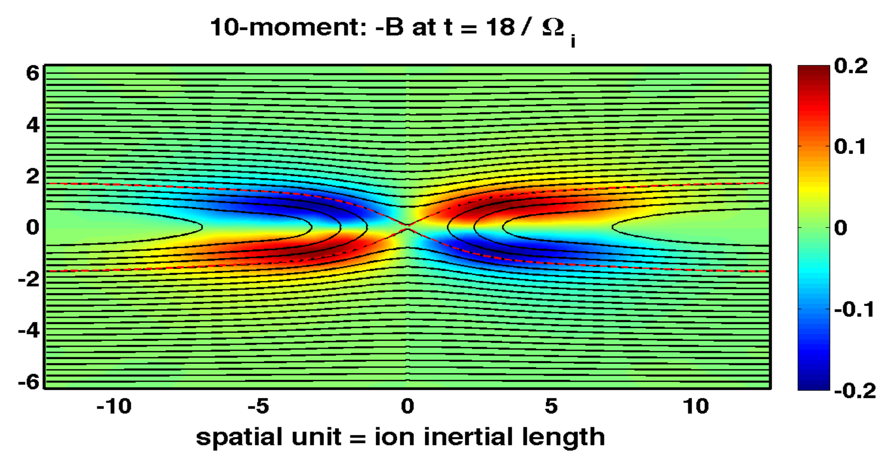


The GEM (Geospace Environment Modeling) magnetic reconnection challenge problem was formulated to compare the ability of different plasma models to resolve the process of magnetic reconnection.

- rectangular domain
- boundary conditions: periodic in the horizontal direction, upper and lower boundaries are conducting walls
- initial conditions: Harris sheet equilibrium perturbed by "pinching" to form an X-point





GEM problem: parameters and boundary conditions

Nondimensionalization. The GEM problem nondimensionalizes time by the ion gyrofrequency $\Omega_i = \frac{eB_0}{m_\mathrm{i}}$ and velocity by the ion Alfvén speed $v_{A,\mathrm{i}} := \frac{B_0}{\mu_0 m_\mathrm{i} n_0}$ (making the nondimensionalized version of the permittivity of free space the reciprocal of the light speed squared).

Computational domain. The computational domain is the rectangular domain $[-L_x/2, L_x/2] \times [-L_y/2, L_y/2]$, where $L_x = 8\pi$ and $L_y = 4\pi$. The problem is symmetric under reflection across either the horizontal or vertical axis.

Boundary conditions. The domain is periodic in the x-axis. The boundaries perpendicular to the y-axis are thermally insulating conducting wall boundaries. A conducting wall boundary is a solid wall boundary (with slip boundary conditions in the case of ideal plasma) for the fluid variables, and the electric field at the boundary has no component parallel to the boundary. We also assume that magnetic field runs parallel to and so does not penetrate the boundary (this follows from Ohm's law of ideal MHD, but we assume it holds generally). So at the conducting wall boundaries

$$\partial_y \rho_\mathrm{s} = 0, \qquad \partial_y B_x = B_y = \partial_y B_z = 0,$$

$$\partial_y u_{\mathrm{s}x} = u_{\mathrm{s}y} = \partial_y u_{\mathrm{s}z} = 0, \qquad E_x = \partial_y E_y = E_z = 0.$$

GEM problem: initial conditions

Initial conditions. The initial conditions are a perturbed Harris sheet equilibrium. The unperturbed equilibrium is given by

$$\mathbf{B}(y) = B_0 \tanh(y/\lambda)\mathbf{e}_x, \qquad p(y) = \frac{B_0^2}{2n_0} n(y),$$

$$n_i(y) = n_e(y) = n_0(1/5 + \operatorname{sech}^2(y/\lambda)), \qquad p_e(y) = \frac{T_e}{T_i + T_e} p(y),$$

$$\mathbf{E} = 0, \qquad p_i(y) = \frac{T_i}{T_i + T_e} p(y).$$

On top of this the magnetic field is perturbed by

$$\delta\mathbf{B} = -\mathbf{e}_z \times \nabla(\psi), \text{ where}$$

$$\psi(x, y) = \psi_0 \cos(2\pi x/L_x) \cos(\pi y/L_y).$$

In the GEM problem the initial condition constants are

$$\lambda = 0.5, \qquad B_0 = 1, \qquad n_0 = 1, \qquad \psi_0 = B_0/10.$$

The GEM problem has been studied using a variety of plasma models.

- **Kinetic** models represent particle velocity $\mathbf{v}$ explicitly.
  - **Vlasov/Boltzmann** models evolve the particle density of each species in phase space, $f_s(\mathbf{x}, \mathbf{v}, t)$.
  - **Particle-in-cell (PIC)** models track individual particles.
- **Fluid** models evolve *moments* of the particle distribution function, which may be taken as parameters of a presumed distribution function.

Fluid models vary in the *number of moments*:

- **Five-moment** models evolve density $\int_{\mathbf{v}} f_s$, momentum $\int_{\mathbf{v}} f_s\mathbf{v}$, and energy $\int_{\mathbf{v}} f_s v^2/2$ and naturally assume a velocity-space distribution nearly *Maxwellian* (isotropic normally distributed).
- **Ten-moment** models evolve density, momentum, and an energy tensor $\int_{\mathbf{v}} f_s\mathbf{vv}$ and naturally assume a velocity-space distribution nearly *Gaussian* (anisotropic normally distributed).

Fluid models vary in the *number of fluids*:

- **Two-fluid** plasma models evolve separate fluid equations for ions and electrons.
- **One-fluid** plasma models (i.e. *magnetohydrodynamics (MHD)*) evolve moments summed over all species. MHD infers electric field from net current balance (assuming quasineutrality), and assumes current from Ampere's law (neglecting displacement current $\partial_t\mathbf{E}$).

We take the Boltzmann model as the "truth" and PIC simulations as attempts to approximate the Boltzmann equation. We desire simple, computationally efficient fluid models that accurately replicate the behavior of kinetic models.

Equations of particle model

The first principles of classical mechanics say that particle positions $\mathbf{x}_\mathrm{p}(t)$ and velocities $\mathbf{v}_\mathrm{p}(t)$ change according to Newton's laws of motion

$$d_t\tilde{\mathbf{v}}_\mathrm{p} = \mathbf{F}_\mathrm{p} \qquad d_t\mathbf{x}_\mathrm{p} = \mathbf{v}_\mathrm{p}$$

and the Lorentz electromagnetic force law

$$\mathbf{F}_\mathrm{p} = \frac{q_\mathrm{p}}{m_\mathrm{p}}\Big(\mathbf{E}|_{\mathbf{x}_\mathrm{p}} + \mathbf{v}_\mathrm{p} \times \mathbf{B}|_{\mathbf{x}_\mathrm{p}}\Big)$$

in response to the electric field $\mathbf{B}(\mathbf{x}, t)$ and magnetic field $\mathbf{E}(\mathbf{x}, t)$, which in turn evolve according to Maxwell's equations

$$\partial_t\mathbf{B} = -\nabla \times \mathbf{E}, \qquad \nabla \cdot \mathbf{B} = 0,$$
$$\partial_t\mathbf{E} = c^2\nabla \times B - \mathbf{J}/\epsilon_0, \quad \nabla \cdot \mathbf{E} = \sigma/\epsilon_0,$$

where the current density $\mathbf{J}(\mathbf{x}, t)$ and charge density $\sigma(\mathbf{x}, t)$ source terms are determined by particle position and velocity

$$\mathbf{J} = \sum_\mathrm{p} S_\mathrm{p}(\mathbf{x}_\mathrm{p}) q_\mathrm{p}\mathbf{v}_\mathrm{p},$$
$$\sigma = \sum_\mathrm{p} S_\mathrm{p}(\mathbf{x}_\mathrm{p}) q_\mathrm{p}.$$

In these equations c is the speed of light, ϵ_0 is electric permittivity, p is particle index, $\tilde{\mathbf{v}}_\mathrm{p}(t) = \gamma_\mathrm{p}\mathbf{v}_\mathrm{p}$ is (proper) particle velocity, where $\gamma = \big(1 - (v/c)^2\big)^{-1/2} \approx 1$ is the Lorentz factor, q_p is particle charge, m_p is particle mass, and $S_\mathrm{p}(\mathbf{x} - \mathbf{x}_\mathrm{p})$ is particle charge distribution (e.g., a unit impulse function).

Equations of Boltzmann/Vlasov model

The Boltzmann equation asserts conservation of particle number density $f_{\mathrm{s}}(\mathbf{x}, \tilde{\mathbf{v}}, t)$ in phase space:

$$\partial_t f_{\mathrm{s}} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} f_{\mathrm{s}}) + \nabla_{\tilde{\mathbf{v}}} \cdot \left(\frac{q_{\mathrm{s}}}{m_{\mathrm{s}}} (\mathbf{E} + \mathbf{v} \times \mathbf{B}) f_{\mathrm{s}} \right) = C_{\mathrm{s}};$$

here $\tilde{\mathbf{v}} = \gamma \mathbf{v} \approx \mathbf{v}$ is (proper) velocity and C_{s} is a collision operator which operates on the function $(\tilde{\mathbf{v}}, \mathrm{p}) \mapsto f_{\mathrm{p}}(t, \mathbf{x}, \tilde{\mathbf{v}})$, where p ranges over all species. The collisionless Boltzmann equation (alias *Vlasov equation*) asserts that $C_{\mathrm{s}} = 0$. The relations

$$\mathbf{J} = \sum_{\mathrm{s}} \int_{\mathbf{v}} f_{\mathrm{s}} q_{\mathrm{s}} \mathbf{v}, \qquad \sigma = \sum_{\mathrm{s}} \int_{\mathbf{v}} f_{\mathrm{s}} q_{\mathrm{s}}.$$

couple the Boltzmann equation to Maxwell's equations

$$\partial_t \mathbf{B} = -\nabla \times \mathbf{E}, \qquad \nabla \cdot \mathbf{B} = 0,$$

$$\partial_t \mathbf{E} = c^2 \nabla \times B - \mathbf{J}/\epsilon_0, \qquad \nabla \cdot \mathbf{E} = \sigma/\epsilon_0.$$

Equations of five-moment two-fluid-Maxwell model

Generic physical equations for the five-moment two-fluid model are: (1) conservation of mass and momentum and pressure evolution for each species:

$$\partial_t \rho_s + \nabla \cdot (\rho_s \mathbf{u}_s) = 0,$$

$$\partial_t (\rho_s \mathbf{u}_s) + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s) + \nabla p_s = \frac{q_s}{m_s} \rho_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \mathbf{R}_s + \nabla \cdot \underline{\underline{\sigma}}_s,$$

$$\partial_t \left(\frac{3}{2} p_s \right) + \nabla \cdot \left(\mathbf{u}_s \frac{3}{2} p_s \right) + p_s \nabla \cdot \mathbf{u} + \nabla \cdot \mathbf{q}_s = \underline{\underline{\sigma}} : \nabla \mathbf{u} + Q_s^f + Q_s^t,$$

and (2) Maxwell's equations for evolution of electromagnetic field:

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0,$$

$$\partial_t \mathbf{E} - c^2 \nabla \times \mathbf{B} = -\mathbf{J}/\epsilon, \qquad \nabla \cdot \mathbf{E} = \sigma/\epsilon.$$

A linear isotropic entropy-respecting viscous stress closure is $\underline{\underline{\sigma}} = 2\mu \left(\mathrm{Sym}\,(\nabla \mathbf{u}) - \nabla \cdot \mathbf{u}\mathbb{I}/3 \right)$. In these 5-moment simulations, however, we neglect all collisional effects. So we neglect viscosity, heat flux ($\mathbf{q}_s = 0$), resistive drag force ($\mathbf{R}_s = 0$), resistive heating ($Q_s^f = 0$), and interspecies thermal equilibration ($Q_s^t = 0$),

Equations of ten-moment two-fluid-Maxwell model

Generic physical equations for the ten-moment two-fluid model are: (1) conservation of mass and momentum and pressure tensor evolution for each species:

$$\partial_t \rho_\mathrm{s} + \nabla \cdot (\rho_\mathrm{s} \mathbf{u}_\mathrm{s}) = 0,$$

$$\partial_t (\rho_\mathrm{s} \mathbf{u}_\mathrm{s}) + \nabla \cdot (\rho_\mathrm{s} \mathbf{u}_\mathrm{s} \mathbf{u}_\mathrm{s} + \mathbb{P}_\mathrm{s}) = \frac{q_\mathrm{s}}{m_\mathrm{s}} \rho_\mathrm{s} (\mathbf{E} + \mathbf{u}_\mathrm{s} \times \mathbf{B}) + \mathbf{R}_\mathrm{s},$$

$$\partial_t \mathbb{P}_\mathrm{s} + \nabla \cdot (\mathbf{u}_\mathrm{s} \mathbb{P}_\mathrm{s}) + 2\,\mathrm{Sym}\,(\mathbb{P}_\mathrm{s} \cdot \nabla \mathbf{u}_\mathrm{s}) + \nabla \cdot \mathbb{q}_\mathrm{s} = \mathbb{R}_\mathrm{s} + \mathbb{Q}_\mathrm{s}^f + \mathbb{Q}_\mathrm{s}^t + 2\,\mathrm{Sym}\,(\frac{q_\mathrm{s}}{m_\mathrm{s}} \mathbb{P}_\mathrm{s} \times \mathbf{B})$$

and (2) Maxwell's equations for evolution of electromagnetic field:

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0,$$

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A linear isotropic entropy-respecting isotropization closure is $\mathbb{R}_\mathrm{s} = \frac{1}{\tau_\mathrm{s}} \left(\frac{1}{3} (\mathrm{tr}\, \mathbb{P}_\mathrm{s}) \mathbb{I} - \mathbb{P}_\mathrm{s} \right)$, where for the isotropization period of species s we used $\tau_\mathrm{s} = \tau_0 \sqrt{\frac{\det \mathbb{P}_\mathrm{s}}{\rho_\mathrm{s}^5}} m_\mathrm{s}^3$, which is based on the Braginskii closure; for the GEM problem this means that $\tau_i / \tau_e \approxeq (m_i/m_e)^{5/4}$. The viscosity is related to the isotropization period by $\mu_\mathrm{s} = p_\mathrm{s} \tau_\mathrm{s}$. We set $\tau_0 = 50$. We neglect all other collisional terms: the heat flux tensors $\mathbb{q}_\mathrm{s}$, the resistive drag forces $\mathbf{R}_\mathrm{s}$, the frictional heating tensors $\mathbb{Q}_\mathrm{s}^f$, and the temperature equilibration tensors $\mathbb{Q}_\mathrm{s}^t$.

Results

We ran 5-moment and 10-moment simulations of the GEM problem and compared the results with the Vlasov simulations of [ScGr06][1] and the PIC simulations of [Pritchett01][2]. [3] All plots were made at the point in time when 16% (representing one nondimensionalized unit) of the magnetic flux initially passing through the positive y-axis had been reconnected.

We find that two-fluid models are able to replicate published kinetic simulation plots fairly well, and that the agreement is better for the ten-moment model than for the five-moment model. In particular, in comparison with kinetic simulations

- the reconnection electric field agrees well for both fluid models,
- the 10-moment model reconnects at about the same rate and the 5-moment model reconnects a bit sooner, and
- the 10-moment models matches the qualitative structure of the diffusion region fairly well.

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[1] [ScGr06] H. Schmitz and R. Grauer, *Kinetic Vlasov simulations of collisionless magnetic reconnection*, Phys. Plasmas 13, 092309 (2006); doi:10.1063/1.2347101

[2] [Pritchett01] P. L. Pritchett, *Geospace Environment Modeling magnetic reconnection challenge: Simulation with a full particle electromagnetic code*, Journal of Geophysical Research, vol. 106, no. A3, pp. 3783–3798 (2001)

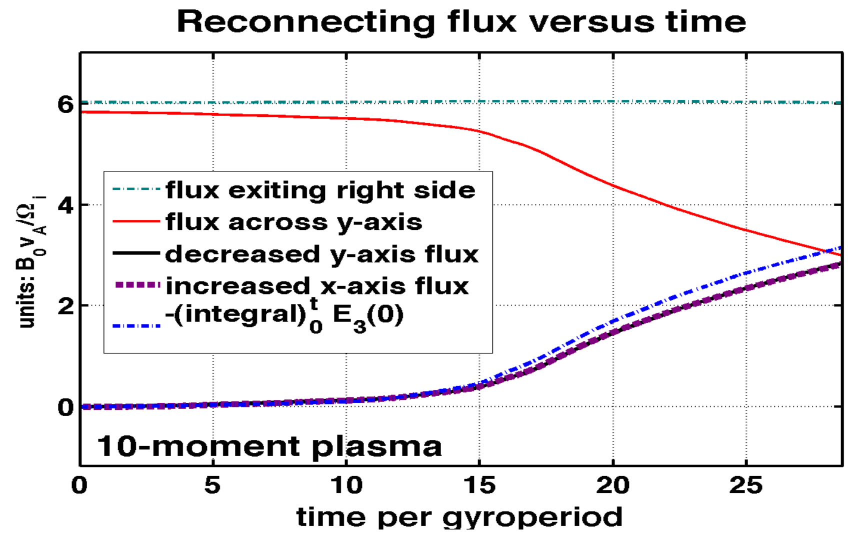
[3] We have to negate some quantities because we call the vertical axis y and the out-of-plane axis z, opposite to the convention of [Pritchett01] and [ScGr06].

Reconnection for 5-moment and 10-moment two-fluid models

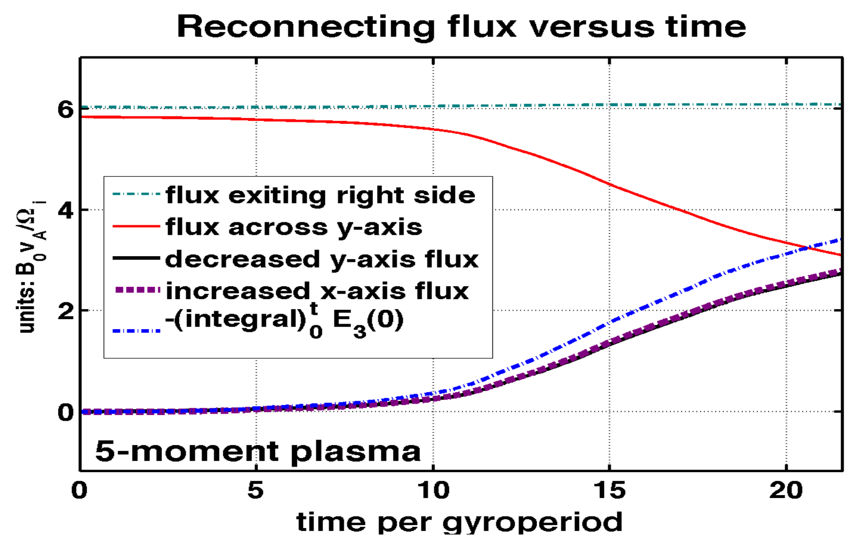
We compared the time until 16% reconnection of the 5-moment and 10-moment models with reported results:

model	16% flux reconnected
Vlasov [ScGr06]	$t = 17.7/\Omega_i$:
PIC [Pritchet01]	$t = 15.7/\Omega_i$:
10-moment	$t = 18/\Omega_i$:
5-moment	$t = 13.5/\Omega_i$:

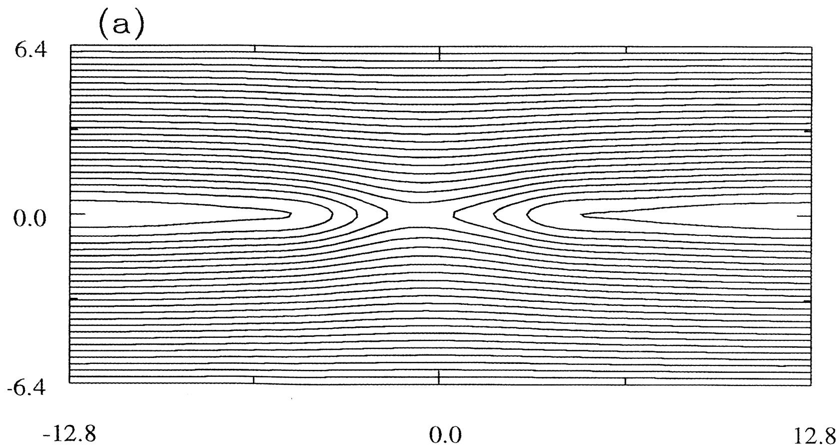
The ten-moment model attained 16% flux reconnected at about $t = 18/\Omega_i$:



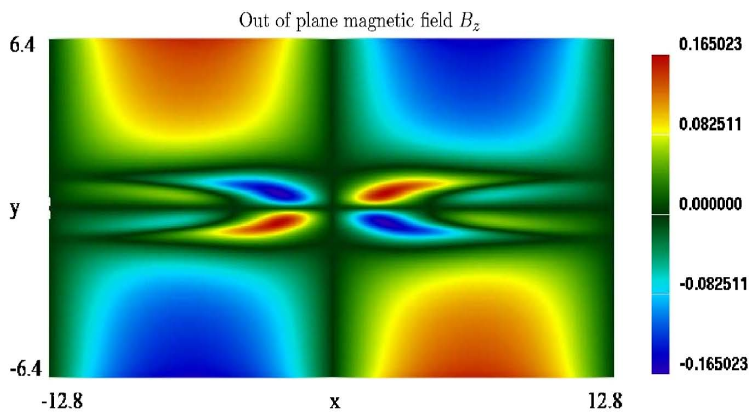
The five-moment model attained 16% flux reconnected at about $t = 13.5/\Omega_i$:



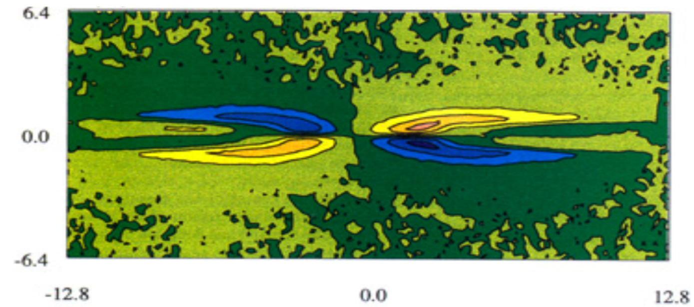
Magnetic field at 16% reconnected



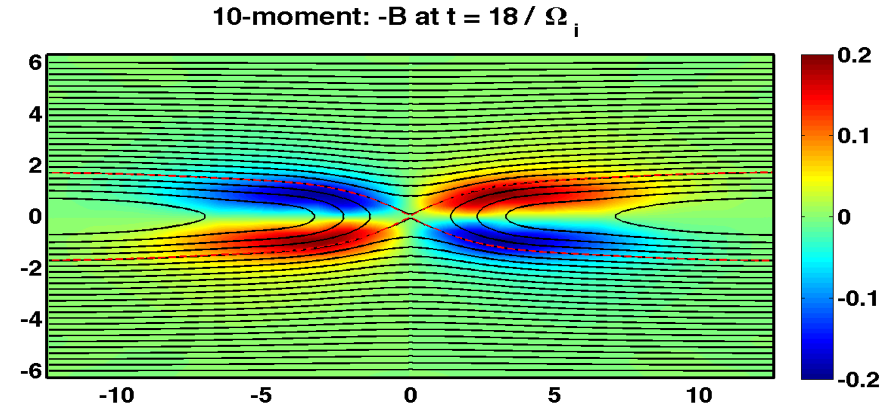
Magnetic field lines for PIC at $\Omega_i t = 15.7$ [Pritchett01]

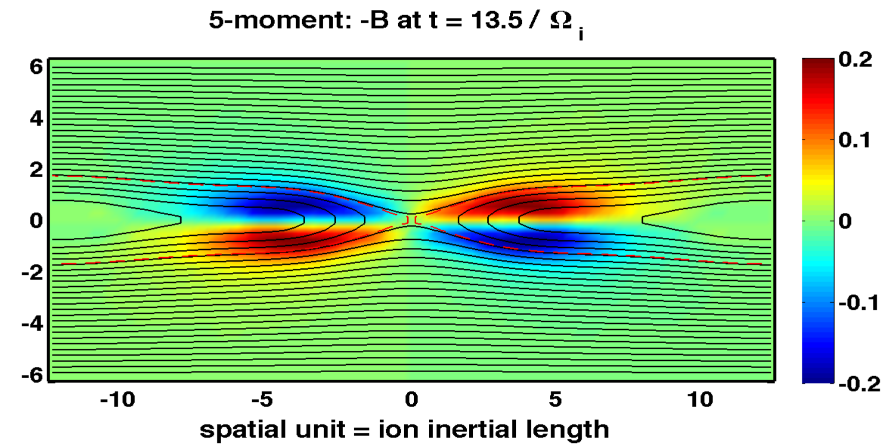


Magnetic field for Vlasov at $\Omega_i t = 17.7$ [ScGr06]

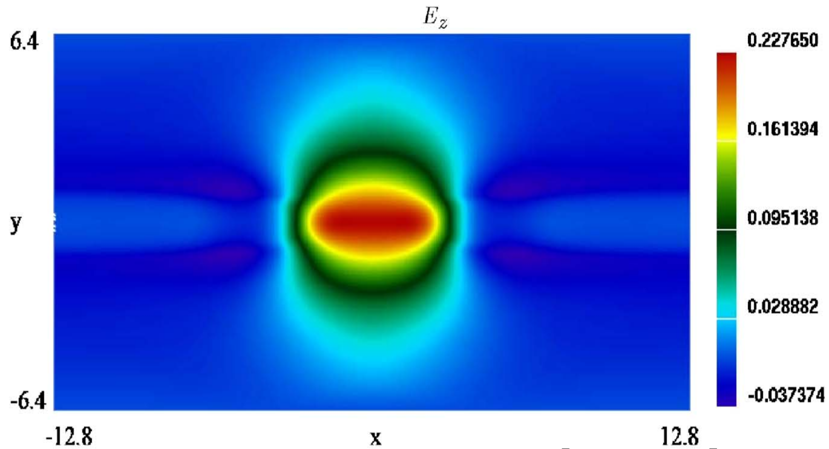


Out-of-plane magnetic field of [Pritchett01]

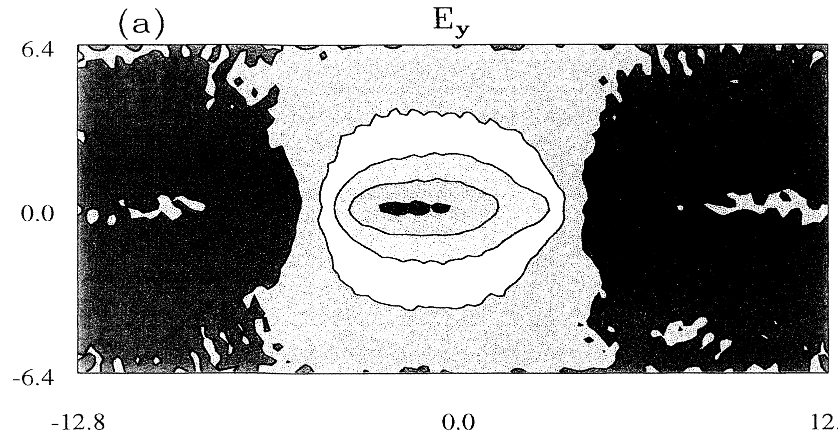




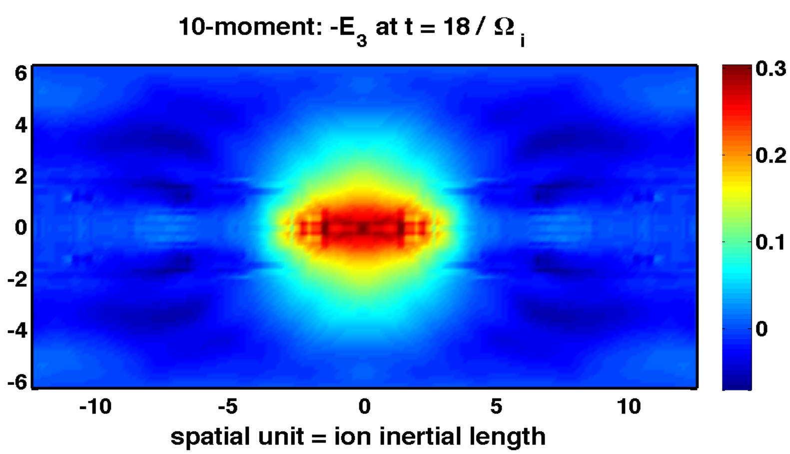
Out-of plane electric field at about 16% reconnected



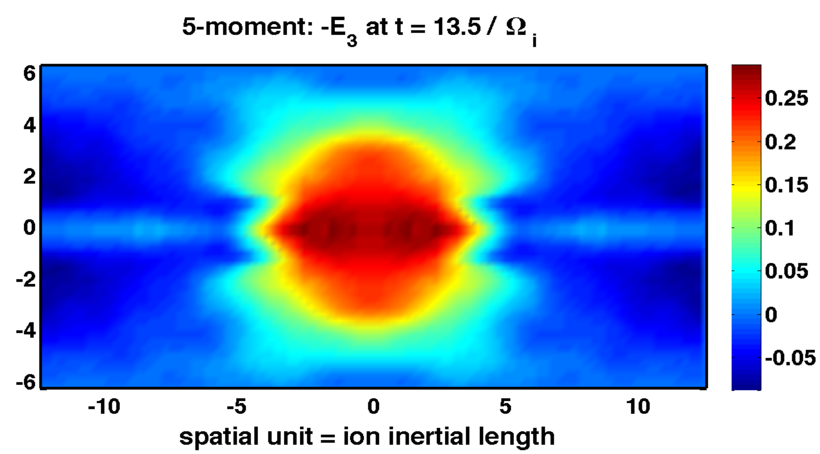
Vlasov at $\Omega_i t = 17.7$ [ScGr06]



PIC averaged over $\Omega_i t = 15.0$ to 15.6 [Pritchett01]

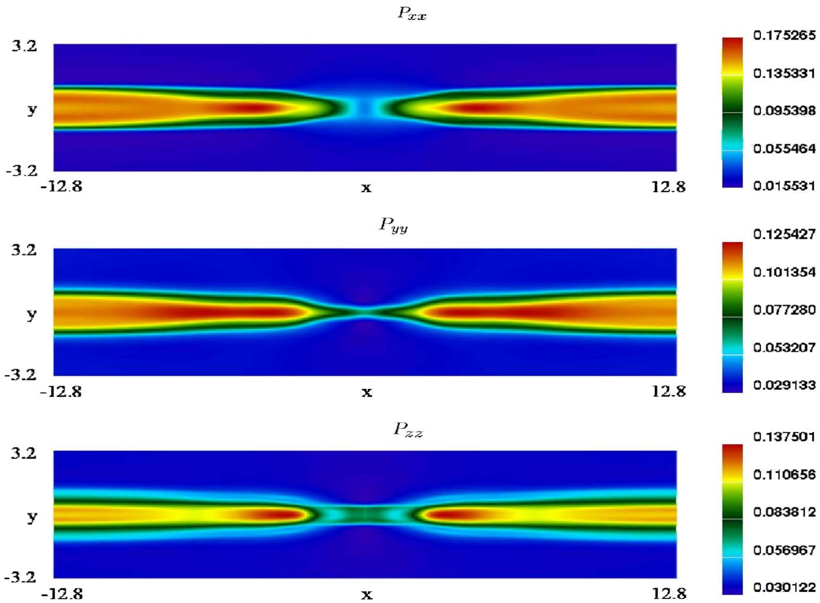


10-moment $\Omega_i t = 18$

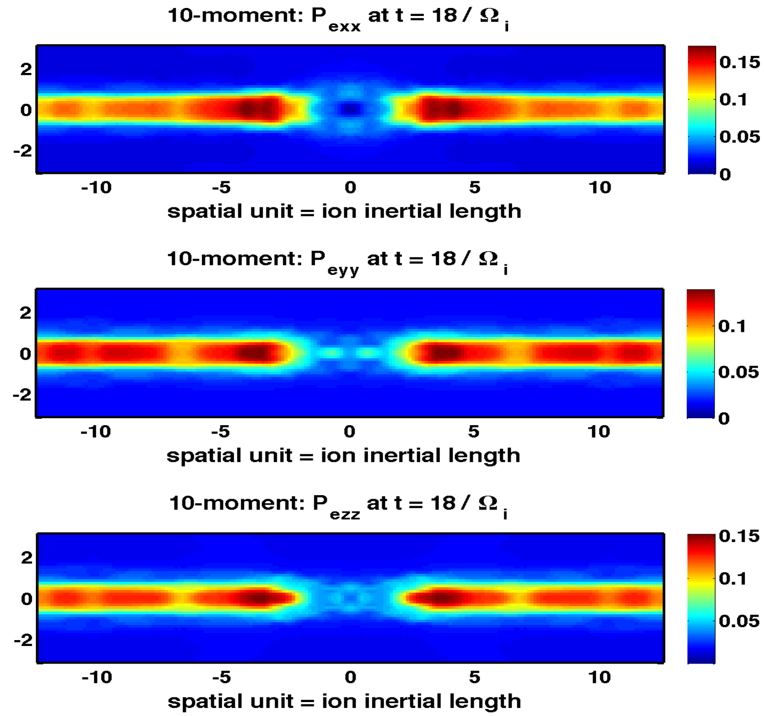


5-moment $\Omega_i t = 13.5$

Diagonal components of the electron pressure tensor

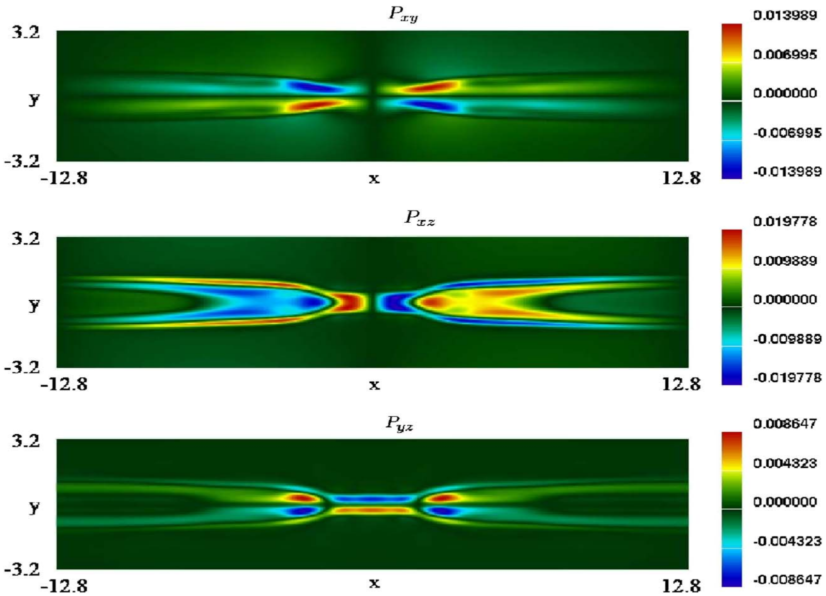


Diagonal components of the electron pressure tensor for Vlasov simulation at $\Omega_i t = 17.7$ [ScGr06]

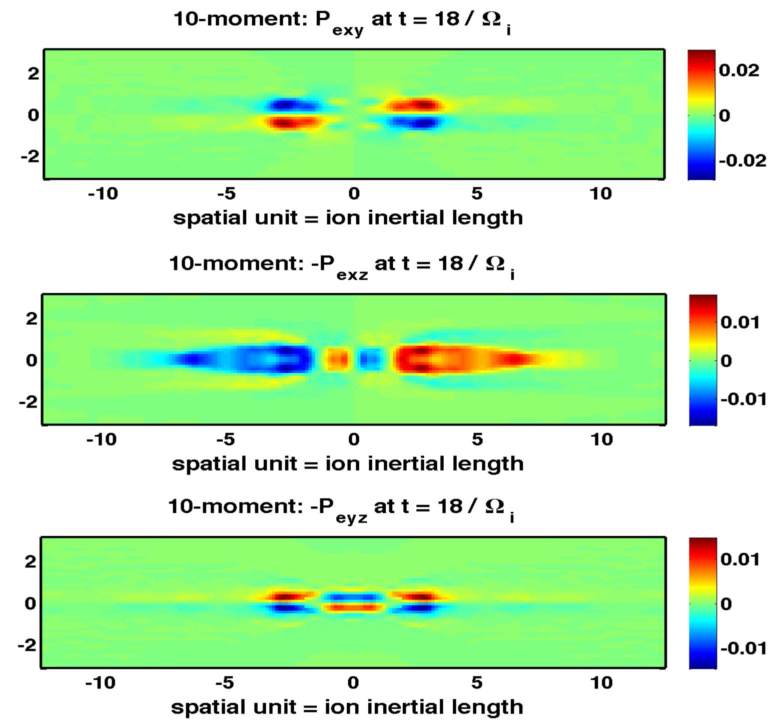


Diagonal components of the electron pressure tensor for 10-moment simulation at $\Omega_i t = 18$

Off-diagonal components of the electron pressure tensor



Off-diagonal components of the electron pressure tensor for Vlasov simulation at $\Omega_i t = 17.7$ [ScGr06]



Off-diagonal components of the electron pressure tensor for 10-moment simulation at $\Omega_i t = 18$

Plasma Theory: GEM problem

There are only three sources that can provide for magnetic reconnection in any plasma model.

At the X-point, "Ohm's law" says that the rate of reconnection is the sum of a *resistive term*, a *nongyrotropic pressure term*, and an *inertial term*:

$$\text{rate of reconnection} = \mathbf{E}_3(0) = \left[\frac{-\mathbf{R}_i}{e n_i} + \frac{\nabla \cdot \mathbb{P}_i}{e n_i} + \frac{m_i}{e} \partial_t \mathbf{u}_i \right]_3 \Bigg|_{\text{origin}}.$$

Consequences:

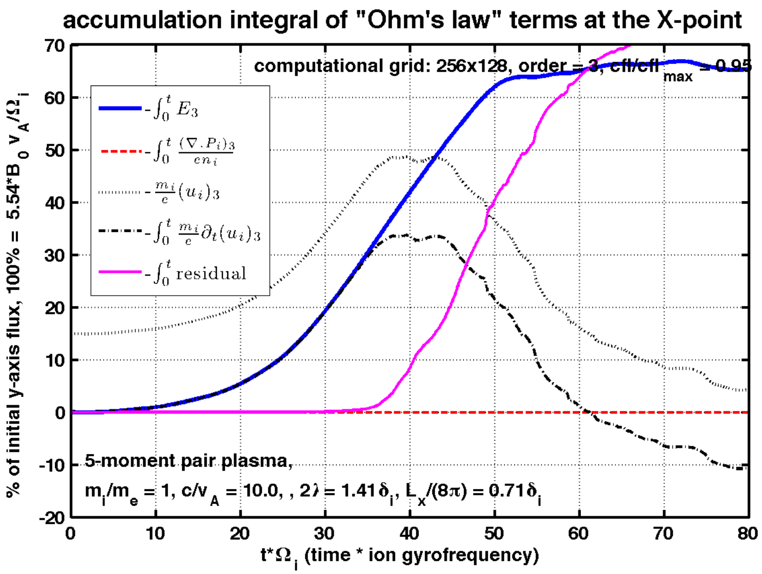
① For *steady-state* reconnection without resistivity the *pressure* term must provide for the reconnection.

② For a *gyrotropic* plasma without resistivity the *inertial* term must provide for the reconnection; i.e. each species velocity at the origin should track exactly with reconnected flux.

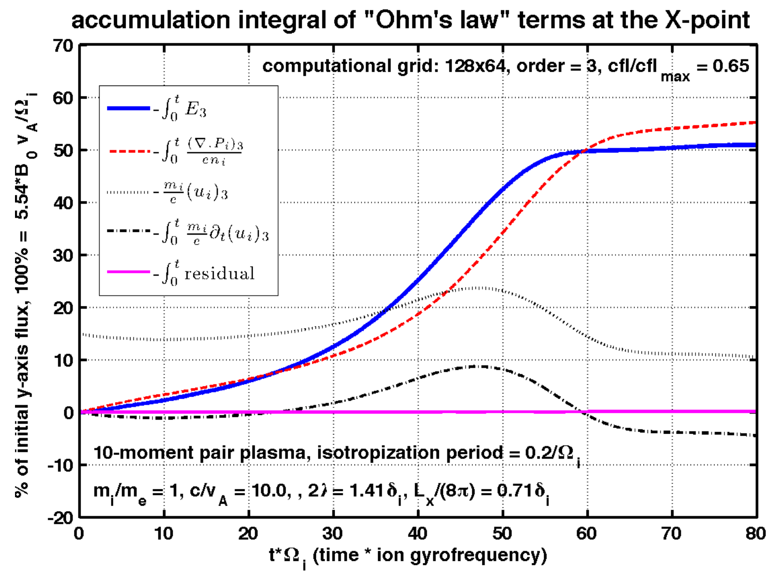
5-moment versus 10-moment reconnection at X-point (pair plasma).

Without viscosity or resistivity entropy cannot change. The current at the X-point is forced to ramp up with reconnected flux as cancelled magnetic field energy is converted to kinetic energy. Eventually numerical viscosity/resistivity kicks in to balance reconnection and numerical entropy production permits steady reconnection.

Five-moment reconnection:



Ten-moment reconnection with relaxation toward isotropy (viscosity)



Convergence challenges

- The GEM problem is unstable and incompletely posed (unless supplemented by a nonvanishing collision model).
  - Reconnection is a relaxation from higher to lower entropy.
  - Collisionless models are hyperbolic (entropy-conserving).
  - So simulations of reconnection in collisionless models (e.g. Vlasov simulations, PIC simulations, and my 5-moment simulations) rely on numerical entropy dissipation.
  - GEM problem is unstable to tearing instability (formation of magnetic islands) and depends critically on choice of collision model.
  - Magnetic islands (plasmoids) tend to form. When enforcing symmetry about the X-point a magnetic island sometimes forms there, stopping reconnection there.
- Problems I am having:
  - **Central islands**: I am enforcing symmetry to facilitate X-point analysis. When a central island forms I get no reconnection. Central islands seem more likely to occur as I refine the mesh.
  - **Negative density/pressure at X-point.** My 10-moment simulations typically blow up at the X-point, generally in the interval $20 \leq \Omega_i t \leq 25$.
    * Near-vanishing density and/or highly anisotropic pressure near X-point causes vulnerability.
    * I hope that adding heat flux diffusion will regularize behavior near the X-point.

Future work

① Add viscosity to five-moment model and verify that it agrees with the 10-moment model with isotropization.

② Add heat flux to regularize solutions and demonstrate convergence for fine mesh.

Acknowledgements

Thanks for my advisor, James Rossmanith, and to Ammar Hakim for helpful conversations.