WEATHER TO FLY

BY DAN GUDGEL

Mountain Wave Parameters

In previous renditions of Weather to Fly, we discussed characteristics and the conceptual model of the mountain wave, mountain wave forecasting, and implications of flight in and around the phenomenon. To close out the topic, a little discussion on the complexity of numerical forecasting is appropriate. When the subject of mountain waves is discussed very often someone eventually asks, "Why does the mountain wave form?" A complete explanation of why the wave forms and a numerical description requires advanced mathematics and physics. This is the reason for the vague answer that is often given regarding the appropriate question "why?" from inquisitive aviators.

The complexity of a comprehensive numerical description for a mountain wave cannot be overstated. In keeping with the meteorology profession's reputation of subterfuge – along with the complexity of atmospheric interactions that lead to the development of the mountain or lee wave – I use this month's installment of Weather to Fly to just give just a taste of the theory. I will point out a few of the assumptions, and numerical equation terms and variables that only begin to describe the atmospheric motion that results in the development of the mountain wave. I am also including a more extensive, yet still abridged, reference section for those who wish to see examples of wave research continually underway. Personal recommendations for further information about mountain wave would be the "The Mountain Wave Project" [7] and any research conducted by the University Corporation for Atmospheric Research (UCAR) and National Center for Atmospheric Research (NCAR).

In looking at the conceptual model of the mountain wave, the definition of the (vertical) wavelength is the distance between the crests of the waves. Reliably this is the distance between the first and second waves (not the distance from the disturbing terrain feature to the first wave crest). The amplitude of a wave is the measure of the air's vertical change in its oscillation (See ***Diagram #1: "Mountain Wave Conceptual Model"***).

Courtesy of Holton [4], a mountain or lee wave develops when air is forced to flow over a mountain under statically stable conditions. Individual air parcels are displaced from a level where they were at an equilibrium level. As a result of the displacement by terrain, the air parcels undergo buoyancy oscillations as they move downstream of the mountain. An internal gravity wave system is excited in the lee of the mountain. A *gravity wave [5]* is defined as a wave disturbance in which buoyancy (or reduced gravity) acts as a restoring force on parcels of

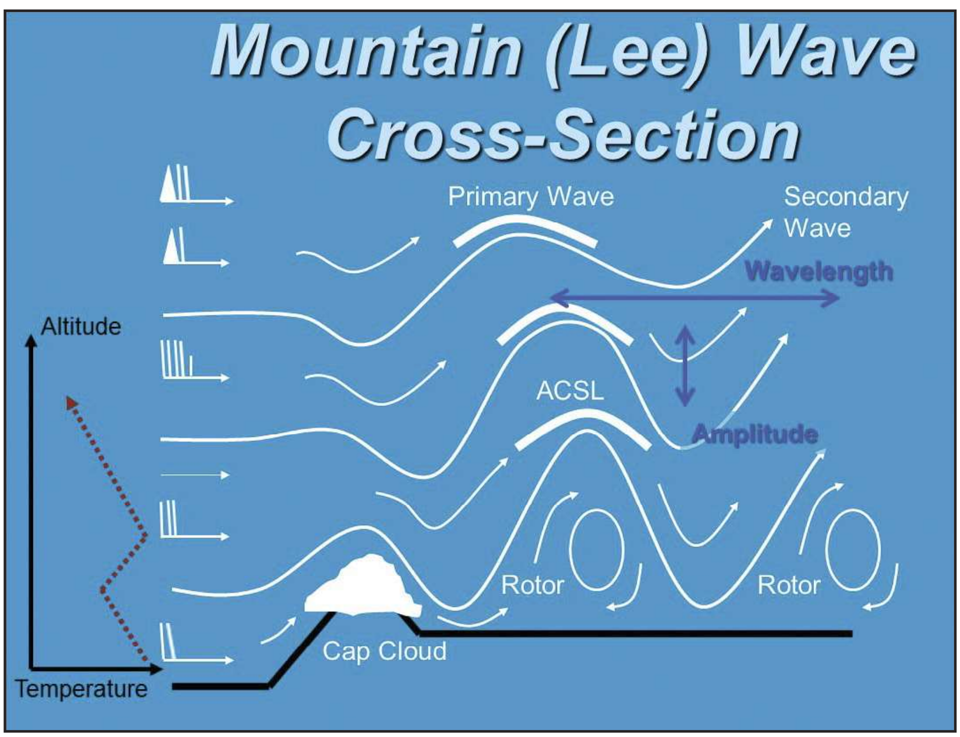
Hydrostatic Equation [5]

Underscoring just some of the assumptions and yet considerations that constitutes the complexity of attempting to describe the motion of the atmosphere in regard to mountain waves, this meteorological formula derivation represents the vertical component of the vector equation of motion. All Coriolis, earth curvature, frictional, and vertical acceleration terms are considered negligible compared with those involving the vertical pressure force and the force of gravity.

Thus $\delta p / \delta z = -\rho g$

where p is the pressure, ρ is the density, g the acceleration of gravity, and z the geometric height. For cyclonic-scale motions the error committed in applying the hydrostatic equation to the atmosphere is less than 0.01%.

NOTE: Strong vertical accelerations in thunderstorms and ***mountain waves*** (editor's emphasis) may be 1% of gravity or more in extreme situations.



air displaced from hydrostatic equilibrium. *Hydrostatic equilibrium [5]* is the state of a fluid (the air) with consistent horizontal surfaces of constant pressure and constant mass (or density). In this equilibrium, a balance exists between the force of gravity acting on the mass of air and the pressure force (Note: Remember pressure changes with altitude height gain or loss). With assumptions, the relationship between the pressure and any geometric height in the atmosphere is defined by the *Hydrostatic Equation* (See ***Text Box #1: Hydrostatic Equation [5]***).

The first term that must be addressed by numerical modelers of the atmosphere is stability, and in the mountain wave case, static stability. *Static Stability [5]*, also called hydrostatic stability or vertical stability, is the ability of air at rest to become either turbulent or laminar due to the effects of buoyancy. A fluid - the air - tending to become or remain turbulent is said to be statically unstable; a fluid tending to become or remain laminar is statically stable. A fluid on the borderline between the previous two (which might remain laminar or turbulent depending on its history) is statically neutral. The most prevalent type of the mountain wave, commonly known as a "trapped wave," typically requires static stability. With the aforementioned basic concepts and definitions, meteorologists begin to numerically describe the atmosphere's stability.

The concept of static stability can also be applied to air not at rest by considering only the buoyant effects and neglecting all other shear and inertial effects of motion. *Shear* and *inertial effects of motion* result in dynamic stability contributions, or the measure of the ability of the air to resist or recover from finite perturbations of what was a steady state condition. However, if any of these other dynamic stability effects is indicative that the flow is dynamically unstable, then the flow will become turbulent regardless of the static stability. In other words, turbulence has a physical priority in the atmosphere when considering all possible measures of air flow stability (e.g., the air is turbulent if any one or more of static, dynamic, inertial, etc., effects indicates instability). Turbulence that forms in statically unstable air will act to reduce or eliminate the instability that caused it by moving less dense air up in height and more dense air down thus creating a neutrally buoyant mixture. Thus, turbulence will tend to decay with time as static instabilities are eliminated in the mixing (unless some outside forcing such as heating of the bottom of a layer of air by contact with the warm ground during a sunny day) continually acts to destabilize the air.

By mathematical derivations and assumptions (See ***Text Box #2: Wavelength Relationship***), the vertical wavelength of the gravity wave excited by zonal flow (westerly flow) over a mountain is *proportional to the zonal wind speed*, *and inversely proportional to the square root of the stability* [4]. Mountain lee waves are stationary with respect to the ground. The initial energy source for disturbing the air flow is the ground and this disturbing energy must be transported vertically. At the same time, the phase velocity relative to the mean wind flow has a downward component. In the mathematical derivation of the wavelength, the constant phase velocity of the wave shows a westward (or upstream) tilt of the wave crest with height. When viewed within a coordinate system moving at the speed of the mean zonal wind, constant phase lines of lee waves set up by westerly flow appear to progress upstream toward the west (the direction from that the wind is coming from).

As mentioned, early wave modeling work proceeded with a series of assumptions to keep the Lee-Wave Equation [8] simplified. It was assumed that the amplitude of the waves is relatively small compared to the wavelength (wavelengths ~6 miles or 10 km), and that the effect of the earth's rotation could be dis-

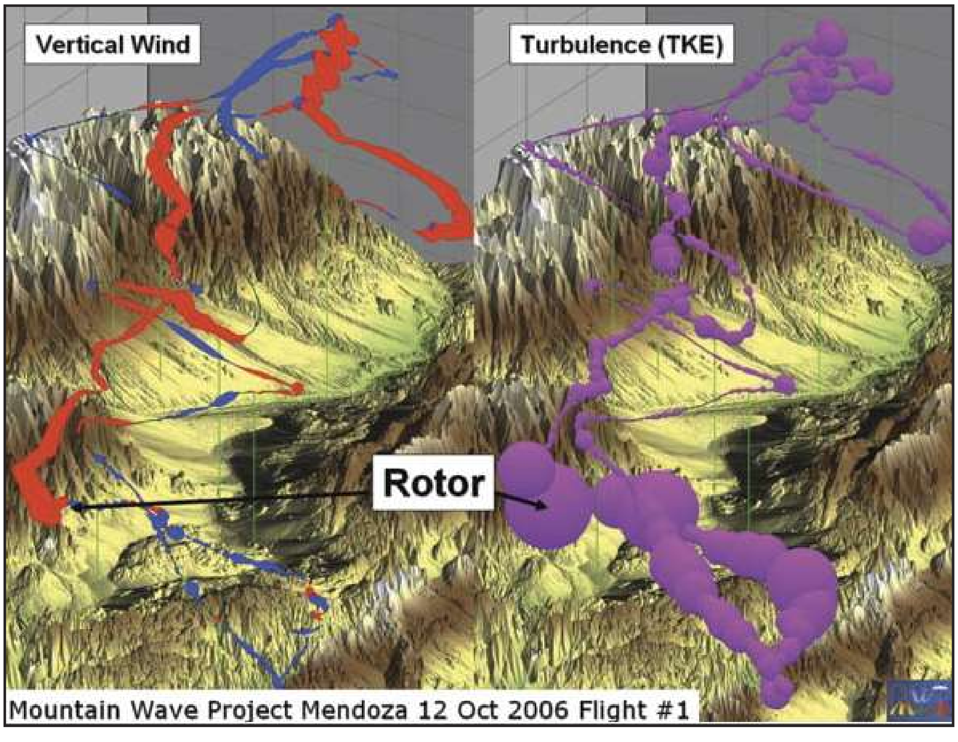
Wavelength Relationship [4]

$$\lambda = [S/u^2 - 1/4H^2]^{1/2} \approx S^{1/2}/u$$

where:

λ = vertical wavelength of the gravity wave;
H = a constant scale height
S = stability; and,
u = zonal wind speed.

By derivations of numerical equations, this relationship shows that the vertical wavelength of the gravity wave excited by zonal flow over a mountain is ***proportional to the zonal wind speed and inversely proportional to the square root of the stability.***



Graphical rendition of vertical wind and turbulence (Turbulent Kinetic Energy) observed in wave. Rendition courtesy of "The Mountain Wave Project" from data gathered during the Terrain-Induced Rotor Experiment in 2006 in the lee of the Southern Sierra Nevada over the Owens Valley.

regarded. The air motion was described in a coordinate system where the wind was relatively undisturbed, and along an axis perpendicular to a mountain ridge considered to have an infinite extension. Other assumptions were that the motion would be described as non-viscous, laminar, and isentropic. *Isentropic* implies that potential temperature is constant with respect to space, in this regard [4].

Turbulent flow is not the type of air movement desired for soaring wave flight. As such, the "trapped wave" regime of air that is relatively stable provides for the laminar flow. Numerical work must account for the effects of both variation of wind and stability with height. Early work by R.S.Scorer and the subsequent development of an older wave forecast tool, the *Scorer Parameter* (See ***Text Box #3: Scorer Parameter [6]***), underscores the importance of *temperature*, *temperature lapse rates*, and *wind shear* in the generation of mountain wave laminar flow. As observed, some degree of stability is desired at lower atmospheric levels with increasing destabilization aloft that often approaches the dry adiabatic lapse rate.

What else makes mountain wave numerical description complex? The basic structure of the mountain wave is initially determined by the size and shape of the mountain. Downwind terrain can interfere with the wave. Constructive interference occurs when the downwind terrain features align favorably within the wavelength to support the updraft of the wave; or destructive interference occurs when the downwind terrain is out of phase with the wavelength. Terrain shape and size must fit in with the functions of the vertical profiles of temperature, wind speed, and moisture in the impinging flow [3] for wave development. Linear theory fits well for the assumption that mountain waves are generated by terrain relatively small compared to the wavelength. If the aforementioned assumption is not the case, then *nonlinear dynamics* play a significantly larger impact on the low-level wave field over the lee slope.

The role of stability as a function of temperature and temperature changes has been discussed. Wind shear is also a key term in the development of the mountain wave. If numerical simulations change only the vertical wind shear, then the following wave development occurs [6]:

- If a wave structure develops that occurs with *weak wind shear* (change in wind speed), on the order of 10 meters/second or 20 knots from mountaintop to the Tropopause (the top of the lowest atmospheric level extending upward from the surface to around 30,000 feet MSL at mid-latitudes in the winter), the waves are primarily in a vertically propagating mode with wave response mostly higher than the mountain ridge. Only minimal disturbed flow is noted downwind of the mountain;
- *Moderate wind shear* with winds increasing 20m/s or 40kts leads to lee waves occurring farther downwind with longer wavelengths aloft. The primary wave has a very pronounced upwind tilt. The mountain wave system then has both high-level vertically propagating and low-level trapped-wave modes. This is an optimum wave condition for pilots looking for maximum altitude or altitude gain; and,
- *Strong wind shear* through the Tropopause, winds increasing 45m/s or 90kts, results in wave energy that is largely trapped in waves in the lower troposphere and minimal disturbed flow at higher altitudes. Wave updrafts develop farther downwind of the mountains.

One other flow structure can develop from terrain influence that is different from the trapped-wave considered above. This type of mountain-wave is referred to as an *atmospheric jump* (or hydraulic jump as studied in engineering and fluid-dynamics). The atmospheric jump is analogous to a shock wave in a compressible fluid. The jump develops one large wave oscillation downwind of the lee slope of a mountain with no resonant waves. Rotor or turbulence forms not only under the wave crest, but also occurs downwind as well. Atmospheric jumps are much less frequent than trapped-wave systems. They tend to favor development with the presence of *high, steep lee slopes*, *strong near-mountain top inversions*, and *relatively weak vertical shear* environments [6].

In summary, accurate and comprehensive numerical descriptions and modeling of a mountain wave (and subsequently the ability to numerically forecast) is quite complex for all aspects of wave development, especially if one is striving for 3-dimensional representation of the wave. The understanding of the complex interactions within the atmosphere has been aided immeasurably by high-speed computing along with technological advances in observation capabilities to the extent that we can graphically display air motion (See ***Diagram #2: "Mountain Wave Project Rotor Depiction [7]"***). In order to model the mountain wave, atmospheric stability and its variation must be defined and measured, any changes in the wind's character (wind speed and direction changes, including eddy development) must be noted and calculated, and the variation of terrain in regard to shape, height, and its influence the initial air flow disturbance must all be numeri-

Scorer Parameter (I^2), [6]

A wave forecast tool that emphasizes the importance of *wind speed*, *stability*, and *shear* throughout the troposphere in the generation of mountain waves:

$$I^2 = [g(\gamma^* - \gamma)/(Tu^2)] - [1/u(d^2u/dz^2)]$$

where;

g = acceleration due to gravity;
γ^* = dry adiabatic lapse rate;
γ = ambient lapse rate of the layer;
T = average temperature in the layer;
u = average wind speed in the layer; and,
d^2u/dz^2 = curvature term, specifically the vertical derivative of the vertical wind shear

If the Scorer Parameter decreases with height, trapped waves are likely. The Scorer Parameter will decrease with height if: *stability decreases with height*, *wind speed increases with height*, and *vertical wind shear increases with height*.

Rules:

- A sharp decrease of I^2 with altitude indicates lee waves; or
- A sharp increase of I^2 with altitude indicates turbulence or rotors.

cally described. And even as the wave is generated, downwind terrain features then interfere with the wave. Given the "introduction to numerical modeling of the mountain wave" in this article and for the sake of my compatriots in the meteorological field, please be a little understanding if we seem elusive when answering questions about "why" a mountain wave forms :) .

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