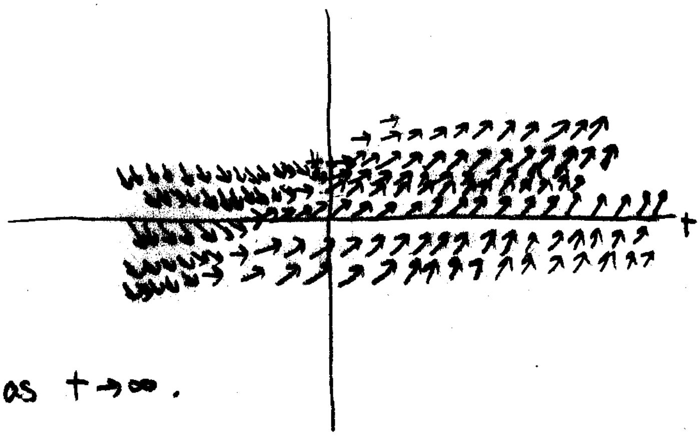
Andy Kaich

Section 2.1, p.39

1) a) $y' + 3y = t + e^{-2t}$

$y' = -3y + t + e^{-2t}$



b) solutions should blow up as $t \to \infty$.

c) $p(t) = 3$

$\int p(t)dt = 3t$

$e^{3t}y' + 3e^{3t}y = te^{3t} + e^{t}$

$\frac{d}{dt}(e^{3t}y) = te^{3t} + e^{t}$

$e^{3t}y = \int te^{3t}dt + \int e^{t}dt$

$e^{3t}y = \frac{1}{3}te^{3t} - \frac{1}{9}e^{3t} + e^{t} + c$

$y(t) = \frac{1}{3}t - \frac{1}{9} + e^{-2t} + ce^{-3t}$

$\int te^{3t}dt = \frac{1}{3}te^{3t} - \int \frac{1}{3}e^{3t}dt$

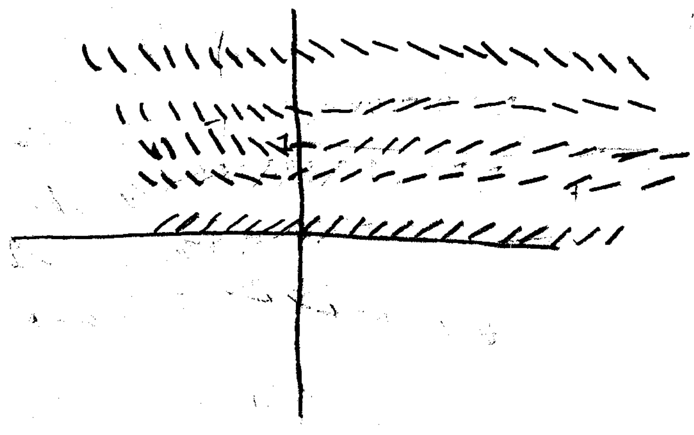
$u = t \quad dv = e^{3t}dt$

$du = dt \quad v = \frac{1}{3}e^{3t}$

$\int te^{3t}dt = \frac{1}{3}te^{3t} - \frac{1}{9}e^{3t} + c$

3) $y' + y = te^{-t} + 1$

a) $y' = -y + te^{-t} + 1$



b) as $t \to \infty$, $y(t) \to 1$.

c) $p(t) = 1 \quad e^{\int p(t)dt} = e^{t}$

$e^{t}y' + e^{t}y = t + e^{t}$

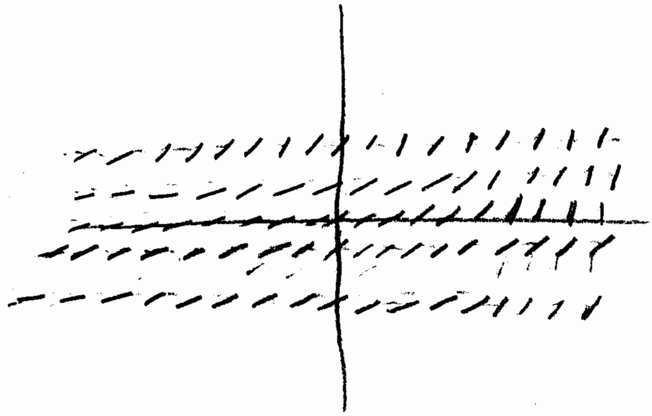
$\frac{d}{dt}(e^{t}y) = t + e^{t}$

$e^{t}y = \frac{1}{2}t^2 + e^{t} + c$

$y(t) = \frac{1}{2}t^2e^{-t} + 1 + ce^{-t}$

5) $y' - 2y = 3e^t$

$y' = 2y + 3e^t$



) $y(t) \to \infty$ as $t \to \infty$

) $p(t) = -2$

$e^{\int p(t)dt} = e^{-2t}$

$e^{-2t} y' - 2e^{-2t} y = 3e^{-t}$

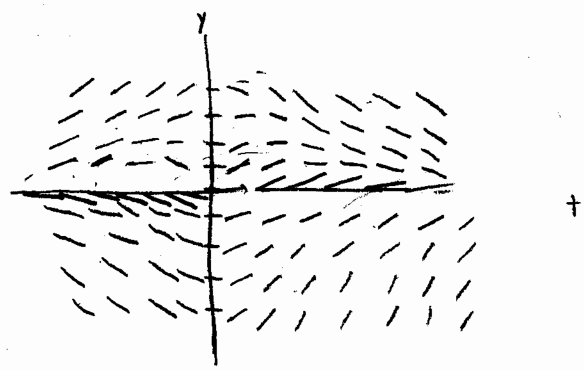
$\frac{d}{dt}\left(e^{-2t} y\right) = 3e^{-t}$

$e^{-2t} y = -3e^{-t} + C$

$y(t) = -3e^t + Ce^{2t}$

7) $y' + 2ty = 2te^{-t^2}$

$y' = -2ty + 2te^{-t^2}$



b) $y(t) \to 0$ as $t \to \infty$

) $p(t) = 2t$

$e^{\int p(t)dt} = e^{t^2}$

$e^{t^2} y' + e^{t^2} \cdot 2ty = 2t$

$\frac{d}{dt}\left(e^{t^2} y\right) = 2t$

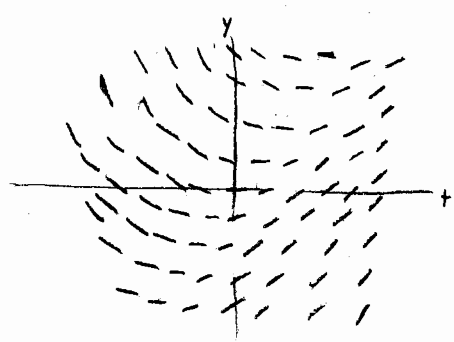
$e^{t^2} y = \int 2t\, dt$

$e^{t^2} y = t^2 + C$

$y(t) = t^2 e^{-t^2} + Ce^{-t^2}$

) $y' + \frac{1}{2} y = \frac{3}{2} t$

$y' = -\frac{1}{2} y + \frac{3}{2} t$



$y(t) \to \infty$ as $t \to \infty$

) $p(t) = \frac{1}{2}$, $e^{\int p(t)dt} = e^{\frac{1}{2} t}$

9c) cont'd

$e^{\frac{1}{2}t}y' + \frac{1}{2}e^{\frac{1}{2}t}y = \frac{3}{2}e^{\frac{1}{2}t}t$

$\frac{d}{dt}\left(ye^{\frac{1}{2}t}\right) = \frac{3}{2}te^{\frac{1}{2}t}$

$ye^{\frac{1}{2}t} = \int \frac{3}{2}te^{\frac{1}{2}t}\,dt$

$ye^{\frac{1}{2}t} = 3te^{\frac{1}{2}t} - 6e^{\frac{1}{2}t} + C$

$\boxed{y(t) = 3t - 6 + Ce^{-\frac{1}{2}t}}$

13) $y' - y = 2te^{2t}, \quad y(0) = 1$

$e^{\int p(t)dt} = e^{-t}$

$\frac{d}{dt}\left(ye^{-t}\right) = 2te^{t}$

$ye^{-t} = \int 2te^{t}\,dt$

$ye^{-t} = 2te^{t} - 2e^{t} + C$

$y(t) = 2te^{2t} - 2e^{2t} + Ce^{t}$

$1 = -2 + C$

$C = 3$

$\boxed{y(t) = 2te^{2t} - 2e^{2t} + 3e^{t}}$

15) $ty' + 2y = t^2 - t + 1 \quad y(1) = \frac{1}{2}$

$y' + \frac{2}{t}y = t - 1 + \frac{1}{t}$

$p(t) = \frac{2}{t} \quad e^{\int p(t)dt} = e^{2\ln t} = t^2$

$t^2y' + 2t = t^3 - t^2 + t$

$\frac{d}{dt}(t^2y) = t^3 - t^2 + t$

$t^2y = \frac{1}{4}t^4 - \frac{1}{3}t^3 + \frac{1}{2}t^2 + C$

$y(t) = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + Ct^{-2}$

$\frac{1}{2} = \frac{1}{4} - \frac{1}{3} + \frac{1}{2} + C$

$C = \frac{1}{12}$

$\boxed{y(t) = \frac{1}{4}t^2 - \frac{1}{3}t + \frac{1}{2} + \frac{1}{12t^2}}$

17) $y' - 2y = e^{2t} \quad y(0) = 2$

$e^{\int p(t)dt} = e^{-2t}$

$e^{-2t}y' - 2e^{-2t}y = 1$

$\frac{d}{dt}\left(ye^{-2t}\right) = 1$

$ye^{-2t} = t + C$

$y(t) = te^{2t} + Ce^{2t}$

$2 = C$

$\boxed{y(t) = te^{2t} + 2e^{2t}}$