Sec. 3.4 p. 164 # 2, 4, 11, 13, 17, 18, 19 Andy Raich

2) $e^{1+2i} = e\,e^{2i} = e(\cos 2 + i\sin 2)$

4) $e^{2-\frac{\pi}{2}i} = e^2 e^{-\frac{\pi}{2}i} = e^2\left(\cos(-\tfrac{\pi}{2}) + i\sin(-\tfrac{\pi}{2})\right) = -ie^2.$

11) $y'' + 6y' + 13y = 0.$

$r^2 + 6r + 13 = 0$

$r = \frac{-6 \pm \sqrt{36-52}}{2} = -3 \pm i2$

$y(t) = C_1 e^{-3t}\cos 2t + C_2 e^{-3t}\sin 2t$

13) $y'' + 2y' + 1.25y = 0$

$r^2 + 2r + 1.25 = 0$

$r = \frac{-2 \pm \sqrt{4-5}}{2} = -1 \pm \frac{i}{2}$

$y(t) = C_1 e^{-t}\cos\tfrac{1}{2}t + C_2 e^{-t}\sin(\tfrac{1}{2}t).$

17) $y'' + 4y = 0$

$r^2 + 4 = 0$

$r = \pm 2i$

$y(t) = C_1\cos 2t + C_2\sin 2t$

$y(0) = 0 = C_1$

$y'(t) = -2C_1\sin 2t + 2C_2\cos 2t$

$y'(0) = 1 = 2C_2 \quad C_2 = \frac{1}{2}$

$y(t) = \frac{1}{2}\sin 2t$

the soln oscillates as t increases.

18) $y'' + 4y' + 5y = 0 \quad y(0) = 1 \quad y'(0) = 0$

$r^2 + 4r + 5 = 0$

$r = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm i$

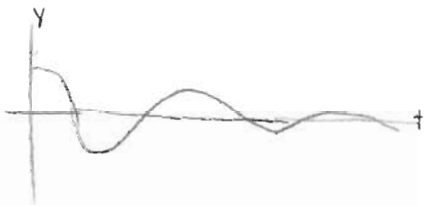
$y(t) = C_1 e^{-2t}\cos t + C_2 e^{-2t}\sin t \qquad 1 = C_1$

$y'(t) = -2C_1 e^{-2t}\cos t - C_1 e^{-2t}\sin t - 2C_2 e^{-2t}\sin t + C_2 e^{-2t}\cos t$

$0 = -2 + C_2 \qquad C_2 = 2$

$y(t) = e^{-2t}\cos t + 2e^{-2t}\sin t$

$y(t) \to 0$ as $t \to \infty$.



19) $y'' - 2y' + 5y = 0 \quad y(\frac{\pi}{2}) = 0 \quad y'(\frac{\pi}{2}) = 2$

$r^2 - 2r + 5 = 0$

$r = \frac{2 \pm \sqrt{4-20}}{2} = 1 \pm 2i$

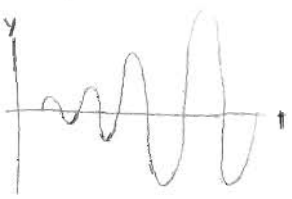
$y(t) = C_1 e^{t}\cos 2t + C_2 e^{t}\sin 2t$

$y'(t) = C_1 e^{t}\cos 2t - 2C_1 e^{t}\sin 2t + C_2 e^{t}\sin 2t + 2C_2 e^{t}\cos 2t$

$y(\tfrac{\pi}{2}) = 0 = -C_1 e^{\pi/2} \Rightarrow C_1 = 0.$

$y'(\tfrac{\pi}{2}) = 2 = -2C_2 e^{\pi/2} \Rightarrow C_2 = -e^{-\pi/2}$

$y(t) = -e^{-\pi/2} e^{t}\sin 2t$



$y(t)$ oscillates with exponentially increasing amplitude as $t \to \infty$.