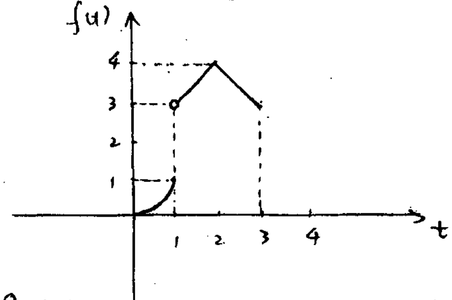
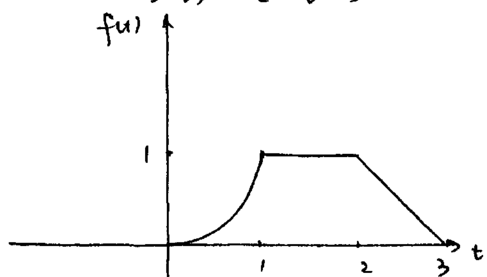
Sec 6.1 P312 By Qian Li

#1 $f(t)=\begin{cases} t^2, & 0\le t\le 1 \\ 2+t, & 1<t\le 2 \\ 6-t, & 2<t\le 3 \end{cases}$



$f(t)$ is piecewise continuous.

#3. $f(t)=\begin{cases} t^2, & 0\le t\le 1 \\ 1, & 1<t\le 2 \\ 3-t, & 2<t\le 3 \end{cases}$



$f(t)$ is continuous

#7. $f(t)=\cosh bt = \frac{1}{2}(e^{bt}+e^{-bt})$

$\mathcal{L}(f(t)) = \frac{1}{2}[\mathcal{L}(e^{bt}) + \mathcal{L}(e^{-bt})]$

$\mathcal{L}(e^{bt}) = \int_0^\infty e^{-st} e^{bt}\, dt$ (assuming $t\ge 0$)

$= \lim_{A\to\infty} \int_0^A e^{-(s-b)t}\, dt$

$\lim_{A\to\infty} \frac{1}{s-b}\left[e^{-(s-b)A} - 1\right]$

$= \frac{1}{s-b}, \quad s>b$

Similarly, $\mathcal{L}(e^{-bt}) = \int_0^\infty e^{-st} e^{-bt}\, dt$

$= \frac{1}{s+b}, \quad s>-b$

Hence

$\mathcal{L}(f) = \frac{1}{2}\left[\frac{1}{s-b} + \frac{1}{s+b}\right], \quad s>|b|$

$= \frac{s}{s^2-b^2}$

#9 $f(t) = e^{at}\cosh bt = e^{at}\,\frac{e^{bt}+e^{-bt}}{2}$

$= \frac{1}{2}\left[e^{(a+b)t} + e^{(a-b)t}\right]$

$\mathcal{L}(e^{(a+b)t}) = \int_0^\infty e^{-st} e^{(a+b)}\, dt$ (assuming $t\ge 0$)

$= \frac{1}{s-(a+b)}, \quad s>(a+b)$

$\mathcal{L}(e^{(a-b)t}) = \int_0^\infty e^{-st}\cdot e^{(a-b)t}\, dt$

$= \frac{1}{s-(a-b)}, \quad s>a-b$

So,

$\mathcal{L}(f) = \frac{1}{2}\left[\mathcal{L}(e^{(a+b)t}) + \mathcal{L}(e^{(a-b)t})\right]$

$= \frac{1}{2}\left[\frac{1}{s-(a+b)} + \frac{1}{s-(a-b)}\right]$

$= \frac{s-a}{(s-a)^2 - b^2}, \quad s>a+|b|$