Section 6.6 #4-11 p.351 Andy Raich

4) Find $\mathcal{L}f(s)$ if $f(t) = \int_0^t (t-\tau)^2 \cos 2\tau \, d\tau$

$$F(s) = \mathcal{L}(t^2)\mathcal{L}(\cos 2t) = \frac{2}{s^3} \cdot \frac{s}{s^2+4} = \boxed{\frac{2}{s^2(s^2+4)}} \quad s>0$$

5) Find $F(s)$ if $f(t) = \int_0^t e^{-(t-\tau)} \sin\tau \, d\tau$

$$F(s) = \mathcal{L}(e^{-t})\mathcal{L}(\sin t) = \frac{1}{s+1} \quad \frac{1}{s^2+1} = \frac{1}{(s+1)(s^2+1)} \quad s>-1$$

6) Find $F(s)$ if $f(t) = \int_0^t (t-\tau)e^{\tau} d\tau$

$$F(s) = \mathcal{L}(t)\mathcal{L}(e^t) = \frac{1}{s^2} \cdot \frac{1}{s-1} = \boxed{\frac{1}{s^2(s-1)}} \quad s>1$$

7) Find $F(s)$ if $f(t) = \int_0^t \sin(t-\tau)\cos\tau \, d\tau$

$$F(s) = \mathcal{L}(\sin t)\mathcal{L}(\cos t) = \frac{1}{s^2+1} \cdot \frac{s}{s^2+1} = \frac{s}{(s^2+1)^2}$$

8) Find $f(t)$ if $F(s) = \frac{1}{s^4(s^2+1)}$.

$$\frac{1}{s^4(s^2+1)} = \frac{1}{s^4} \cdot \frac{1}{(s^2+1)} = \mathcal{L}\left(\frac{t^3}{3!}\right)\mathcal{L}(\sin t), \quad \boxed{f(t) = \int_0^t \frac{(t-\tau)^3}{3!} \sin\tau \, d\tau}$$

9) Find $f(t)$ if $F(s) = \frac{s}{(s+1)(s^2+4)}$.

$$\frac{s}{(s+1)(s^2+4)} = \frac{1}{s+1} \cdot \frac{s}{s^2+4} = \mathcal{L}(e^{-t})\mathcal{L}(\cos 2t), \quad \boxed{f(t) = \int_0^t e^{-(t-\tau)} \cos 2\tau \, d\tau}$$

10) Find $f(t)$ when $F(s) = \frac{1}{(s+1)^2(s^2+4)}$.

$$\frac{1}{(s+1)^2(s^2+4)} = \frac{1}{(s+1)^2} \cdot \frac{1}{s^2+4} = \mathcal{L}(te^{-t})\mathcal{L}\left(\frac{\sin 2t}{2}\right); \quad \boxed{f(t) = \int_0^t (t-\tau)e^{-(t-\tau)}\left(\tfrac{1}{2}\sin 2\tau\right) d\tau}$$

11) Find $f(t)$ if $F(s) = \frac{G(s)}{s^2+1}$.

$$\frac{G(s)}{s^2+1} = G(s) \cdot \frac{1}{s^2+1} = \mathcal{L}(g(t))\mathcal{L}(\cos t); \quad \boxed{\int_0^t g(t-\tau)\sin\tau \, d\tau = f(t)}$$