

5. Ausgabe

$$\frac{d^4 y}{dt^4} + 81y = 0 \quad y(1) = e^t$$

5.15

$$r^4 + 81 = 0$$

$$r^4 = -81$$

$$r = R e^{i\theta}$$

$$\Rightarrow R^4 e^{i4\theta} = -81 e^{i\pi}$$

$$= 81 e^{i3\pi}$$

$$= 81 e^{i\pi}$$

$$= 81 e^{i3\pi}$$

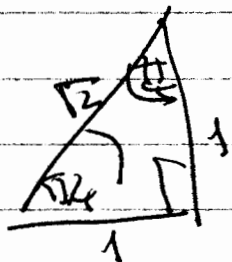
$$R = 3$$

$$\theta_1 = \pi/4$$

$$\theta_2 = 3\pi/4$$

$$\theta_3 = 5\pi/4$$

$$\theta_4 = 7\pi/4$$



$$\Rightarrow r_1 = 3 e^{i\pi/4}$$

$$= 3 \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$$

$$= 3 \left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right)$$

$$(5.15) \quad r_2 = 3 e^{i3\pi/4}$$

$$= 3 \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$$

$$= 3 \left(-\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} \right)$$

$$r_3 = 3 e^{i5\pi/4}$$

$$= 3 \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right)$$

$$= 3 \left(-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)$$

$$r_4 = 3 e^{i7\pi/4}$$

$$= 3 \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

$$= 3 \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}} \right)$$

$$y_1(x) = e^{\frac{3}{\sqrt{2}}t} \cos \left(\frac{3t}{\sqrt{2}} \right), \quad y_4(x) = e^{\frac{3}{\sqrt{2}}t} \sin \left(\frac{3t}{\sqrt{2}} \right)$$

$$(5.15) \quad y_2(x) = e^{-\frac{3}{\sqrt{2}}t} \cos \left(\frac{3t}{\sqrt{2}} \right), \quad y_3(x) = e^{-\frac{3}{\sqrt{2}}t} \sin \left(\frac{3t}{\sqrt{2}} \right)$$

$$y(x) = c_1 e^{\frac{3}{\sqrt{2}}t} \cos \left(\frac{3t}{\sqrt{2}} \right) + c_2 e^{\frac{3}{\sqrt{2}}t} \sin \left(\frac{3t}{\sqrt{2}} \right)$$

$$+ c_3 e^{-\frac{3}{\sqrt{2}}t} \cos \left(\frac{3t}{\sqrt{2}} \right) + c_4 e^{-\frac{3}{\sqrt{2}}t} \sin \left(\frac{3t}{\sqrt{2}} \right)$$

(15.15)