Answer

4 (a) $y''' - y'' + y' - y = 0 \quad y(0)=1, y'(0)=0, y''(0)=-1$

$y(t) = e^{rt} \qquad r^3 - r^2 + r - 1 = 0$ (5pts)

guess a root : $r=1$ works

$$r-1 \;\big|\; r^3 - r^2 + r - 1 \qquad (r^2 + 1)$$

$$r^3 - r^2$$

$$r - 1$$

$$r - 1$$

So $(r^3 - r^2 + r - 1) = (r-1)(r^2+1)$

$\Rightarrow r_1 = 1, \; r_2 = i, \; r_3 = -i$

$\Rightarrow y_1(t) = e^t, \; y_2(t) = \cos t, \; y_3(t) = \sin t$ (5pts)

$y(t) = c_1 e^t + c_2 \cos t + c_3 \sin t$

$y'(t) = c_1 e^t - c_2 \sin t + c_3 \cos t$

$y''(t) = c_1 e^t - c_2 \cos t - c_3 \sin t$

$y(0) = 1 \Rightarrow \quad 1 = c_1 + c_2$

$y'(0) = 0 \Rightarrow \quad 0 = c_1 + c_3 \quad \Rightarrow c_1 = -c_3$

$y''(0) = -1 \Rightarrow \quad -1 = c_1 - c_2$ (20pts total)

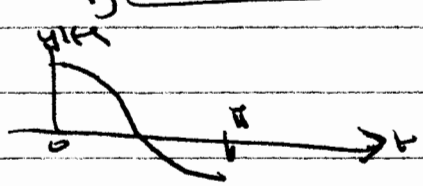
$1 = c_1 + c_2$

$-1 = c_1 - c_2$

$0 = 2c_1 \Rightarrow c_1 = 0 \Rightarrow c_2 = 1 \Rightarrow c_3 = 0$

$\boxed{y(t) = \cos t}$ (5pts)

(b)



graph of cost (5pts)