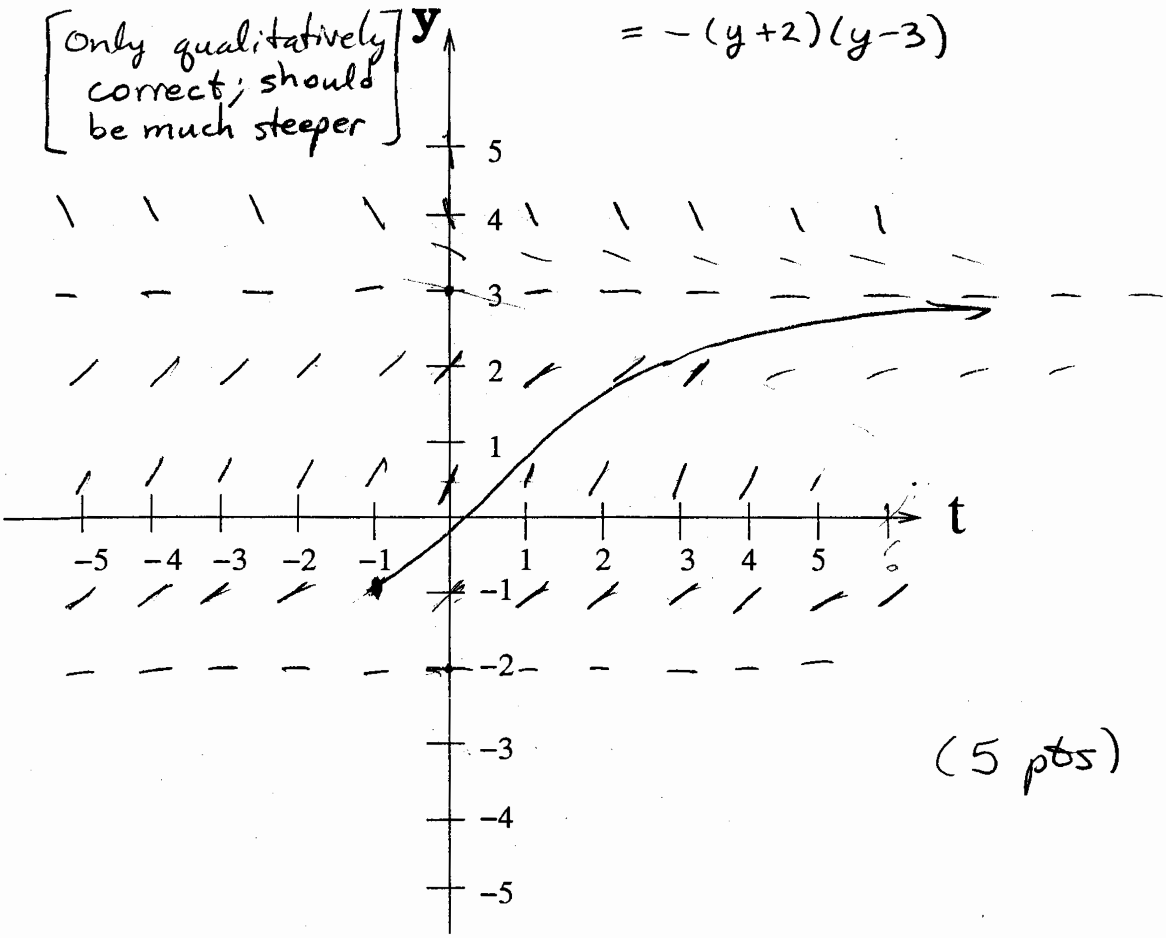
1. (a) Draw the directions fields for $-5 \leq y \leq 5, \quad -5 \leq t \leq 5$ for the differential equation

$$\frac{dy}{dt} = 6 + y - y^2 = -(y^2 - y - 6) = -((y - \tfrac{1}{2})^2 - 6\tfrac{1}{4})$$
$$= -((y - \tfrac{1}{2})^2 - 6\tfrac{1}{4})$$
$$= -(y+2)(y-3)$$



(5 pts)

(b) What is limit as $t \to \infty$ for the solution to initial value problem with $y(-1) = -1$? (You don't have to solve the equation, just give the answer using part (a).)

ANS. 3

(5 pts)

2. Solve the initial value problem

$$\frac{dy}{dt} = \frac{2t}{t^2+1}y + t + 1, \quad y(0) = 1.$$

ANS.

$$y = (t^2+1)\left[\tfrac{1}{2}\ln(t^2+1) + \arctan t + 1\right]$$

$$y' + \underbrace{\left(\frac{-2t}{t^2+1}\right)}_{\text{Call } p(t)} y = t+1$$

Integrating factor:

$$\int p(t)\,dt = \int \frac{-2t}{t^2+1}\,dt$$

$$= -\ln(t^2+1)$$

$$\mu(t) = e^{\int p(t)\,dt} = e^{-\ln(t^2+1)}$$

(8 pts) $$= \frac{1}{t^2+1}$$

(8 pts) $$\left[y\left(\frac{1}{t^2+1}\right)\right]' = \frac{t+1}{t^2+1}$$

$$y\left(\frac{1}{t^2+1}\right) = \int \frac{t}{t^2+1}\,dt + \int \frac{dt}{t^2+1}$$

(8 pts) $$= \tfrac{1}{2}\ln(t^2+1) + \arctan t + C$$

Use $y(0) = 1$ to find C:

(5 pts) $$\boxed{1 = C}$$

(1 pt) $$y = (t^2+1)\left[\tfrac{1}{2}\ln(t^2+1) + \arctan t + 1\right]$$