Slemrod March 31, 2005

NAME:	 	
TA's NAME:		

Calculators Okay!

Problem	Score
1	
2	
3	
4	
Total	

SHOW YOUR WORK!

1. Use Laplace transforms to solve

$$y'' + 9y = g(t), \ y(0) = 0, \ y'(0) = 1,$$

where

$$g(t) = \begin{cases} \frac{t}{6}, & 0 \le t < 12 \\ 2, & 12 < t \end{cases}$$

	$\mathbf{x} + \mathbf{c}$
^	

(20 pts)

2. Solve using Laplace transforms

$$2y'' + y' + 4y = \delta(t - \frac{\pi}{6})\sin t,$$

$$y(0) = 0, y'(0) = 0.$$

 $(\sin\frac{\pi}{6} = \frac{1}{2}, \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2})$ and δ is the unit impulse function.

(30) pts)
	, P (2)

3. Find the Laplace transform of the given function

$$\int_0^t \sin(t-\tau)\cos\tau d\tau.$$

ANS.		
		(10 pts)
	·	

4. (a) Use the eigenvalue – eigenvector method to find the general solution of

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ & \\ 4 & -2 \end{bmatrix} \mathbf{x}.$$

ANS.

(20 pts)

(b) Use the result of part (a) to solve the initial value problem with

$$\mathbf{x}(0) = \begin{bmatrix} 2 \\ -3 \end{bmatrix}.$$

ANS.

(10 pts)

(c) Use the result of part (a) to plot a few trajectories in the x_1, x_2 plane. Use arrows to denote increasing t.

