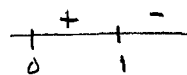


Andy Raich Sec. 9.7 p.556 #1,3,8

$$1) \frac{dr}{dt} = r^2(1-r^2) \quad \frac{d\theta}{dt} = 1$$

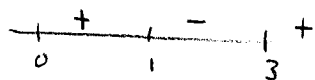


$r=0, 1$

$\theta = t + t_0$

$r=1$ asymptotically stable limit cycle.

$$3) \frac{dr}{dt} = r(r-1)(r-3) \quad \frac{d\theta}{dt} = 1 \quad \theta = t + t_0$$



$r=1$ asymptotically stable limit cycle

$r=3$ unstable periodic sol'n.

$$8) a) \frac{dx}{dt} = -y + x \frac{f(r)}{r} \quad \frac{dy}{dt} = x + y \frac{f(r)}{r}$$

Let r_0 be a zero of $f(r)$ (i.e. $f(r_0) = 0$). Then $\frac{dx}{dt} = -y$, $\frac{dy}{dt} = x$,

so by the chain rule, $\frac{dy}{dx} = \frac{-x}{y}$, so $y dy = -x dx$
 $\frac{1}{2} y^2 = -\frac{1}{2} x^2 + C$

$$x^2 + y^2 = 2C, \text{ a circle, hence}$$

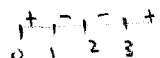
a periodic sol'n. The path is counterclockwise.

$$b) f(r) = r(r-2)^2(r^2-4r+3) = r(r-2)^2(r-3)(r-1) \quad \theta = t + t_0$$

$r=1$ asymptotically stable

$r=2$ semistable limit cycle.

$r=3$ unstable periodic sol'n



(alternatively for a) if $x = r \cos \theta$, $\frac{d\theta}{dt} = 1$, then $\frac{dx}{dt} = \frac{dr}{dt} \cos \theta - r \sin \theta \frac{d\theta}{dt}$
 $y = r \sin \theta$

$$\frac{dy}{dt} = \frac{dr}{dt} \sin \theta + r \cos \theta \frac{d\theta}{dt}$$

$$\frac{dx}{dt} = -y + \frac{r \cos \theta}{r} \frac{dr}{dt} = -y + \frac{x}{r} \frac{dr}{dt}$$

$$\Rightarrow \frac{dr}{dt} = f(r)$$

$$\frac{dy}{dt} = x + \frac{r \sin \theta}{r} \frac{dr}{dt} = x + \frac{y}{r} \frac{dr}{dt}$$

$f(r_0) = 0$ means $\frac{dr}{dt} = 0$,
circular motion