Andy Raich Sec. 9.7 p.556 #1,3,8

1) $\frac{dr}{dt} = r^2(1-r^2)$ $\frac{d\theta}{dt} = 1$

$+$ $-$
0 1

$r = 0, 1$ $\theta = t + t_0$ $r = 1$ asymptotically stable limit cycle.

3) $\frac{dr}{dt} = r(r-1)(r-3)$ $\frac{d\theta}{dt} = 1$ $\theta = t + t_0$

$+$ $-$ $+$
0 1 3

$r = 1$ asymptotically stable limit cycle

$r = 3$ unstable periodic sol'n.

8) a) $\frac{dx}{dt} = -y + x\frac{f(r)}{r}$ $\frac{dy}{dt} = x + y\frac{f(r)}{r}$

Let r_0 be a zero of $f(r)$ (i.e. $f(r_0) = 0$). Then $\frac{dx}{dt} = -y$, $\frac{dy}{dt} = x$,

so by the chain rule, $\frac{dy}{dx} = \frac{-x}{y}$, so $y\,dy = -x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + c$

$x^2 + y^2 = 2c$, a circle, hence

a periodic sol'n. The path is counterclockwise.

b) $f(r) = r(r-2)^2(r^2-4r+3) = r(r-2)^2(r-3)(r-1)$ $\theta = t + t_0$

$+$ $-$ $-$ $+$
0 1 2 3

$r = 1$ asymptotically stable

$r = 2$ semistable limit cycle.

$r = 3$ unstable periodic sol'n

(alternatively for a) if $x = r\cos\theta$, $y = r\sin\theta$, $\frac{d\theta}{dt} = 1$, then $\frac{dx}{dt} = \frac{dr}{dt}\cos\theta - r\sin\theta\frac{d\theta}{dt}$

$\frac{dy}{dt} = \frac{dr}{dt}\sin\theta + r\cos\theta\frac{d\theta}{dt}$

$\frac{dx}{dt} = -y + \frac{r\cos\theta}{r}\frac{dr}{dt} = -y + \frac{x\,dr/dt}{r}$

$\frac{dy}{dt} = x + \frac{r\sin\theta}{r}\frac{dr}{dt} = x + \frac{y\,dr/dt}{r}$

$\Rightarrow \frac{dr}{dt} = f(r)$

$f(r_0) = 0$ means $\frac{dr}{dt} = 0$, circular motion