Maxwell's Equations

$J = \rho v$

$\rho_t + \nabla\cdot J = 0$

SI units

$$\begin{cases} \nabla\cdot E = \dfrac{\rho}{\varepsilon_0} \\ \nabla\cdot B = 0 \\ \nabla\times E = -B_t \\ \nabla\times B = \mu_0 J + \mu_0\varepsilon_0 E_t \end{cases} \Leftrightarrow \left(c^2\,\nabla\times B = \frac{J}{\varepsilon_0} + E_t\right)$$

Let $c^2 = \dfrac{1}{\mu_0\varepsilon_0}$

$$c\,\nabla\times(cB) = \frac{J}{\varepsilon_0} + E_t$$

$J = \rho v \Rightarrow \frac{J}{\varepsilon_0} = \left(\frac{\rho}{\varepsilon_0}\right)v$

HL (Heaviside-Lorentz)

i.e.

$$\begin{cases} \nabla\cdot E = \left(\dfrac{\rho}{\varepsilon_0}\right) & \nabla\cdot E = \rho_\ell \\ \nabla\cdot(cB) = 0 & \nabla\cdot B_\ell = 0 \\ c\,\nabla\times E = -(cB)_t & c\,\nabla\times E = -(B_\ell)_t \\ c\,\nabla\times(cB) = \left(\dfrac{J}{\varepsilon_0}\right) + E_t & c\,\nabla\times B_\ell = J_\ell + E_t \end{cases}$$

where

$$\boxed{\begin{aligned} \rho_\ell &:= \frac{\rho}{\varepsilon_0} = \rho_n = 4\pi\rho_g \\ J_\ell &= \frac{J}{\varepsilon_0} = cJ_n = 4\pi J_g \\ B_\ell &= cB = B_n = B_g \end{aligned}}$$

nondimensional units (ND)

$J = \rho v$

$\Rightarrow \left(\frac{J}{c\varepsilon_0}\right) = \left(\frac{\rho}{\varepsilon_0}\right)\left(\frac{v}{c}\right)$

$\frac{\rho}{\varepsilon_0}$

Let $\tilde t = ct$. $(t_n := \tilde t)$

So $E_t = cE_{\tilde t}$

$(cB)_t = c(cB)_{\tilde t}$

i.e.

$$\begin{cases} \nabla\cdot E = \left(\dfrac{\rho}{\varepsilon_0}\right) & \nabla\cdot E = \rho_n \\ \nabla\cdot(cB) = 0 & \nabla\cdot(B_n) = 0 \\ \nabla\times E = -(cB)_{(ct)} & \nabla\times E = -(B_n)_{t_n} \\ \nabla\times(cB) = \left(\dfrac{J}{c\varepsilon_0}\right) + E_{(ct)} & \nabla\times B_n = J_n + (E_n)_{t_n} \end{cases}$$

where

$$\boxed{\begin{aligned} \rho_n &:= \frac{\rho}{\varepsilon_0} = \rho_\ell = 4\pi\rho_g \\ J_n &:= \frac{J}{c\varepsilon_0} = \frac{J_\ell}{c} = 4\pi\frac{J_g}{c} \\ B_n &:= cB = B_\ell = B_g \\ t_n &:= ct = t = t \\ m_n &:= mc^2 \\ v_n &:= \frac{v}{c} \\ A_n &:= cA \end{aligned}}$$

Gaussian units (G)

i.e.

$$\begin{array}{ll} \nabla\cdot E = 4\pi\left(\dfrac{\rho}{4\pi\varepsilon_0}\right) & \nabla\cdot E = 4\pi\rho_g \\ \nabla\cdot(cB) = 0 & \nabla\cdot B_g = 0 \\ c\,\nabla\times E = -(cB)_t & c\,\nabla\times E = -(B_g)_t \\ c\,\nabla\times(cB) = 4\pi\left(\dfrac{J}{4\pi\varepsilon_0}\right) + E_t & c\,\nabla\times B_g = 4\pi J_g + E_t \end{array}$$

where

$$\boxed{\begin{aligned} \rho_g &= \frac{\rho}{4\pi\varepsilon_0} = \frac{\rho_\ell}{4\pi} = \frac{\rho_n}{4\pi} \\ J_g &= \frac{J}{4\pi\varepsilon_0} = \frac{J_\ell}{4\pi} = \frac{cJ_g}{4\pi} \\ B_g &= cB = B_\ell = B_g \end{aligned}}$$

SI again

$$\boxed{\begin{aligned} \rho &= \varepsilon_0\rho_\ell = \varepsilon_0\rho_n = 4\pi\varepsilon_0\rho_g \\ J &= \varepsilon_0 J_\ell = \varepsilon_0 c J_n = 4\pi\varepsilon_0 J_g \\ B &= \tfrac{1}{c}B_\ell = \tfrac{1}{c}B_n = \tfrac{1}{c}B_g \\ t &= t = \tfrac{1}{c}t_n = t. \end{aligned}}$$

Lorentz force law

SI: $F = q(E + v\times B) = m\dot v$

HL: $= q\left(E + \left(\frac{v}{c}\right)\times(cB)\right) = q\left(E + \frac{v}{c}\times B_\ell\right)$

ND: $= q\left(E + \left(\frac{v}{c}\right)\times(cB)\right) = q(E + v_\ell\times B_\ell) = (mc^2)\frac{d\left(\frac{v}{c}\right)}{d(tc)}$

G: $= q\left(E + \frac{v}{c}\times(cB)\right)$

A: $= q(E + v\times B)$

Alec electrostatic units

i.e.

$$\begin{cases} \nabla\cdot E = \left(\dfrac{\rho}{\varepsilon_0}\right) & \nabla\cdot E = \rho_a \\ \nabla\cdot B = 0 & \nabla\cdot B = 0 \\ \nabla\times E = -B_t & \nabla\times E = -B_t \\ c^2\,\nabla\times B = \left(\dfrac{J}{\varepsilon_0}\right) + E_t & c^2\,\nabla\times B = J_a + E_t \end{cases}$$

where

$$\boxed{\begin{aligned} \rho_a &= \frac{\rho}{\varepsilon_0} \\ J_a &= \frac{J}{\varepsilon_0} \end{aligned}}$$

nondimensional potential

$B = \nabla\times A$

$(cB) = \nabla\times(cA)$

$E = -\nabla\varphi - A_t$

$= -\nabla\varphi - (cA)_{(ct)}$

Gaussian potential

$(cB) = \nabla\times(cA)$

$E = -\nabla\varphi - \frac{1}{c}(cA)_t$

Electromagnetic Units

$$\begin{cases} \nabla\cdot B = 0 \\ \nabla\cdot E = k_1\rho \\ E_t = k_2\nabla\times B - k_3 J \\ B_t = -k_4\nabla\times E \end{cases}$$

(Thinking of each law as a proportionality.)

- Eliminate superfluous constants.

Require $\rho_t + \nabla\cdot J = 0$.

$$k_1\rho_t = \nabla\cdot E_t = -k_3\nabla\cdot J$$

So $\boxed{k_3 = k_1}$

- Express k_4 in terms of k_2 & wave speed c.

Suppose $J = 0$, $\rho = 0$.

Then
$$E_{tt} = k_2\nabla\times B_t = -k_2k_4\nabla\times\nabla\times E = k_2k_4\nabla^2 E$$

So $c^2 = k_2k_4$. Let $K_2 = \frac{k_2}{c}$, $k_1 = k_1$

So $k_4 = \frac{c^2}{k_2} = \frac{c}{K_2}$

Now have:

$$\boxed{\begin{aligned} \nabla\cdot B &= 0 \\ \nabla\cdot E &= k_1\rho \\ E_t &= ck_2\nabla\times B - k_1 J \\ B_t &= -\frac{c}{k_2}\nabla\times E \end{aligned}}$$

system	k_1	k_2
SI	$\frac{1}{\varepsilon_0}$	c
HL	1	1
G	4π	1

Rewrite for conversion of results

$$\nabla\cdot(k_2B) = 0$$
$$\nabla\cdot E = (k_1\rho)$$
$$E_t = c\nabla\times(k_2B) - (k_1J)$$
$$(k_2B)_t = -c\nabla\times E$$

Let
$$\boxed{\begin{aligned} \tilde{B} &:= k_2B \\ \tilde{\rho} &:= k_1\rho \\ \tilde{J} &:= k_1J \end{aligned}}$$

These are H-L quantities.

$$\nabla\cdot\tilde{B} = 0$$
$$\nabla\cdot E = \tilde{\rho}$$
$$E_t = c\nabla\times\tilde{B} - \tilde{J}$$
$$\tilde{B}_t = -c\nabla\times\tilde{E}$$

Need to be consistent with Lorentz:

$$\dot{v}m = q\left(E + \frac{v}{c}\times\tilde{B}\right)$$

$$\boxed{\dot{v}m = q\left(E + \frac{v}{c}\times(k_2B)\right)}$$